Quantification of true displacement using apparent displacement along an arbitrary line on a fault plane

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ARTICLE INFO

Article history:
Received 31 December 2007
Received in revised form 24 November 2008
Accepted 4 December 2008
Available online 10 December 2008

Keywords:
Fault
True displacement
Apparent displacement
Quantification

ABSTRACT

This paper introduces some approaches to determine the true displacement ($S_t$) using an apparent displacement ($S_a$) measured from an arbitrary line on a fault plane. The considered parameters are the pitch of slip lineation ($\gamma$), the pitch of a cutoff ($\beta$), the apparent displacement along the observation line ($S_m$), and the pitch of the observation line on the fault plane ($\phi$). We analyzed the following cases. First, if the apparent displacement is taken as the true displacement, the degree of overestimation or underestimation of the true displacement can be calculated. The displacement cannot be obtained along the null line because the pitch of the observation line ($\phi$) is equal to the pitch of the cutoff of the marker ($\beta$). Second, the total true displacement can be obtained not only along the slip direction but also along another particular line depending on the values of $\gamma$ and $\beta$. Third, if the apparent displacements from two non-parallel markers can be measured, the slip direction can be estimated. We apply the methods to calculate the extensions due to the normal faults of San Miguelito in Mesa Central, Mexico. The results indicate that the largest fault strain reaches ca. 0.50 and the smallest fault strain is ca. 0.08. Also, the isolated faults show more regular strain profiles along the fault strikes than the faults with overlapping or intersecting geometries.

1. Introduction

The term “displacement” is an ambiguous word in geology (Tearpock and Bischke, 2003). According to Walsh and Watterson (1988), displacement refers to the displacement accumulated through the whole active period of the fault. This definition indicates that displacement is a total slipp or total true displacement. Displacement also represents the variation in position of a marker displaced by the fault movement (Tearpock and Bischke, 2003). In the light of this concept, displacement is an apparent displacement. Previous work did not distinguish a true displacement from an apparent displacement (e.g. Dawers et al., 1993; Clark and Cox, 1996). In this paper, we use the term “true displacement” ($S_t$) by following Walsh and Watterson’s definition. Therefore, the true strike displacement ($S_{st}$) refers to the component of $S_t$ along the fault strike. True dip displacement ($S_{sd}$) refers to the component of $S_t$ along the fault dip (Fig. 1).

Three problems influence the obtainment of the true displacement. First, observed sections in outcrops may not be vertical at times, and the sample lines may not be perpendicular to the strikes of faults. Second, the beds are not horizontal or the strikes of the beds are not parallel to that of the fault. Third, faults are not absolute dip-slip or strike-slip faults. For all cases above, it is necessary to establish a quantitative relationship between the true displacement and apparent displacements.

Traditionally, the main parameters to determine the fault displacement are slickenside lineations and kinematic indicators on or near the fault (e.g. Billings, 1972; Suppe, 1985; Doblas et al., 1997a, b). Recently, there has been some work for quantitatively determining the fault true displacement (e.g. Rouby et al., 2000; Xu et al., 2004a; Xu et al., 2007). Billi (2003) analyzed the components of fault slip and separations generated by cleavage-controlled fault zone contraction, on the assumption that shortening occurs perpendicularly to solution cleavages. The methods by Xu et al. (2004a) are appropriate only for the faults on subsurface maps. The approaches by Xu et al. (2007) consider only data measured from cross-section perpendicular to the fault strike or from map view. These methods need more assumptions than the approaches that we introduce here.

In this paper we quantify the magnitude of true displacement and the direction of fault slip on faults. The approaches introduced here can be applied to data measured along arbitrary lines on the fault plane, which are more general than methods proposed by Xu et al. (2007).

This paper consists of two parts. The first part is to establish equations for obtaining the magnitude of true displacement, the direction of fault slip, or both, according to the available data. The second part gives an example of how to calculate the strain due to faulting. In most cases, the accurate strain is difficult to obtain if the
true fault slips are not known. The example in this paper provides an excellent application based on our methods.

### 2. Calculations of the true displacement

In order to define the pitch angles, the following conventions are used.

(a) The angle of pitch is in the range from 0° to 90°. Starting from the strike line, the angle is measured in a sense which is down the dip of the plane. This is used in conjunction with conventions b and c.

(b) Direction of pitch is the direction of the strike from which the angle of pitch is measured.

(c) Opposite direction of pitch refers to the direction of strike which is opposite to the strike from which the angle of pitch is measured.

For calculation, four parameters should be known: the pitch of slip lineation (γ), the pitch of a cutoff (β) of a marker (bed, vein, etc.), the pitch of an observation line (ϕ), the apparent displacement along the observation line (S_app). To calculate the magnitude of true displacement, the following cases can be considered (Table 1).

1. **Case of slickenside lineation with opposite pitch direction to that of marker traces**

   There are some principles for determining whether a slickenside lineation and a marker trace or an observation line on the fault has the same or opposite pitch direction (Xu et al., 2007). Sometimes, the pitch of intersection line of a bed with a fault plane could not be directly obtained. This problem can be resolved using the equations (Eqs. A1 to A18) established by Xu et al. (2007). For example, if a fault with an angle of α intersects a marker with an arbitrary marker angle with an angle of θ, the pitch angle of the cutoff (β) will be β = \( \arctan \left( \frac{\tan \alpha \cdot \tan \theta}{\sin \alpha \cdot \sin \theta} \right) \), where μ is the acute intersection angle of the fault strike with the marker strike.

2. **Case of slickenside lineation with opposite pitch direction to marker trace on the fault**

   For this situation, we should consider two sub-cases (Table 1) according to the pitch direction of an observation line on the fault plane (Fig. 1). The first sub-case is that the observation line has the same pitch direction as the slickenside lineation. Let the pitch angle of the line be ϕ. In Fig. 1, for the triangle CC'E, we can obtain

   \[ \angle \text{CC'F} = 90° - \beta, \]
   \[ \angle \text{CC'E} = 90° - \gamma \]
   \[ \therefore \angle \text{CC'E} = \beta + \gamma, \]

   where \( \gamma \) indicates angle.

   For the triangle FCC', FC'C = S_app because \( \angle \text{FCE} = 90° - \gamma \). Therefore, we can infer that \( \angle \text{FCE} = \angle \text{FCC'E} = \phi - \gamma \). By using the Law of Sines of triangle, the following equation can be established

   \[ S_1 = \frac{S_{\text{app}} \sin(180°-\phi + \beta)}{\sin \gamma + \beta} \]

   The condition \( S_{\text{app}} \) holds for \( \sin(\phi + \beta) = \sin(\gamma + \beta) \) and \( \sin(180° - (\phi + \beta)) = \sin(\gamma + \beta) \) reflecting periodicity of trigonometric function; then, we obtain

   \[ \phi_1 = \gamma \]

   and

   \[ \phi_1 = 180° - \gamma - 2\beta \]

   Eqs. (3) and (4) indicate that if \( \phi_1 = \gamma \), or \( \phi_1 = 180° - \gamma - 2\beta \), the measured displacement is equal to the true displacement. For the given values of β and γ (marker and slickenside pitches), the true displacement can be obtained along two lines of observation. One of them is not the slickenside direction. A special case is \( \beta + \gamma = 90° \), in that case \( S_{\text{app}} \) and \( S_1 \) are equal to each other for only one value of \( \phi_2 \). For example, curve 4 in Fig. 2a has two intersection points with true displacement \( S_1 \). At point a, \( \phi_2 \) is equal to 90°, and at point b, \( \phi_2 \) is equal to 80°.

From Eq. (2), given two of the three angles \( \beta, \gamma, \) and \( \phi_2 \), the relationship between \( S_{\text{app}} \) and the third angle can be calculated (Fig. 2b). It can be seen that the value of \( S_{\text{app}} \) (apparent displacement along the observation line) could be larger, equal, or smaller than the total true displacement. From Fig. 2a, we can see that there are two curve tendencies between \( S_{\text{app}} \) and \( \phi_2 \). Small value of \( \phi_2 \), the value of \( S_{\text{app}} \) has a negative relationship with the value of \( \phi_2 \), whereas for large value of

### Table 1

<table>
<thead>
<tr>
<th>Case 1: slickenside lineation with pitch direction opposite to that of marker traces</th>
<th>Case 2: slickenside lineation with the same pitch direction as that of marker traces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1a: the observation line with the same pitch direction as the slickenside lineation.</td>
<td>Case 2a: the observation line with the same pitch direction as that of the slickenside lineation.</td>
</tr>
<tr>
<td>Case 1b: the observation line with the opposite pitch direction to that of the slickenside lineation.</td>
<td>Case 2a-b: ( \beta &gt; \gamma )</td>
</tr>
<tr>
<td>Case 2a-b: ( \beta &gt; \gamma )</td>
<td>Case 2b: the observation line with the opposite pitch direction to that of the slickenside lineation.</td>
</tr>
<tr>
<td>Case 2b-b: ( \beta &gt; \gamma )</td>
<td>Case 2b-b: ( \beta &gt; \gamma )</td>
</tr>
</tbody>
</table>
of the same value of measured displacement (b) show relationship between three types of curves. When slickenside lineation has opposite pitch direction to marker trace on the fault. The is assumed to be equal to 100 with no unite. The results are based on Eq. (2) to the case

\[ S_t = \frac{S_{\text{true}} \sin(90 - \beta)}{\sin(\gamma + \beta)} \]

(5)

The second sub-case is that the measured line has opposite pitch/para direction to the slickenside lineation. In this case, JC is the apparent displacement \( S_{\text{m}} \) in Fig. 1. For the triangle CC', \( \angle CJC = \phi_2 - \beta \), \( \angle CJC = \beta + \gamma \), then \( \angle CC' = 180 - \phi_2 - \gamma \). By using the Law of Sines of triangle, we obtain

\[ S_t / \sin(\phi_2 - \beta) = S_{\text{m}} / \sin(\beta + \gamma) \]

(6)

This equation is only for \( \phi_2 > \beta \). Similarly, for \( \phi_2 < \beta \), we have

\[ S_t = \frac{S_{\text{m}} \sin(\beta - \phi_2)}{\sin(\gamma + \beta)} \]

(7)

For \( S_{\text{m}} = S_t \) in Eq. (6), the following equation can be obtained

\[ \phi_2 = 2\beta + \gamma \]

(8)

From Eqs. (8) and (9), giving different values of \( \beta \) and \( \gamma \), we can calculate the pitch \( \phi_2 \) of the line in which we can measure the true displacement. Note that it is not the slip direction. This is useful, because when the true displacement along the slip direction cannot be measured due to outcrop problem or any other problems, we can obtain the true displacement by measuring along a line obtained from Eqs. (8) and (9).

From Eqs. (6) and (7), by assuming a values of \( S_{\text{m}} \) and giving two arbitrary values from the values of \( \beta \), \( \gamma \), and \( \phi_2 \), the tendency of \( S_{\text{m}} \) can be shown in Fig. 3. From Fig. 3a, we can obtain the following results. First, when \( \phi_2 \) is smaller than \( \beta \), the value of \( S_{\text{m}} \) has positive relationship with the value of \( \phi_2 \), whereas, when \( \phi_2 \) is larger than \( \beta \), the value of \( S_{\text{m}} \) has negative relationship. Second, for \( \phi_2 < \beta \) the value of \( S_{\text{m}} \) is always larger than the true displacement \( S_t \). For \( \phi_2 > \beta \), some measured values \( (S_{\text{m}}) \) are larger than the true displacement \( S_t \). When \( \phi_2 = 50^\circ \), the values of \( S_{\text{m}} \) approach the true displacement \( S_t \). Third, when the value of \( \phi_2 \) approaches the value of \( \beta \), the value of \( S_{\text{m}} \) tend to have infinite value. This produces large difference between the values of \( S_{\text{m}} \) and \( S_t \). The observation line with this value of \( \phi_2 \) is consistent with the null line of Redmond (1972). Along the null line the fault displacement cannot be observed. Fig. 3b shows that for \( \beta = 60^\circ \) and \( \gamma = 30^\circ \), the value of \( \gamma \) has positive relationship with the value of \( S_{\text{m}} \), whereas for \( \gamma = 90^\circ \), the value of \( \gamma \) has negative relationship with the value of \( S_{\text{m}} \). On the other hand, there are coincided curves in Fig. 3b. For example, the curves for \( \gamma = 90^\circ \) and \( \gamma = 20^\circ \) coincide completely. In Fig. 3c, when \( \beta \) is smaller than \( \phi_2 \), the value of \( S_{\text{m}} \) has a positive relationship with the value of \( \phi_2 \), whereas, when \( \beta \) is larger than \( \phi_2 \), the value of \( S_{\text{m}} \) has a negative relationship. The null line is \( \beta = 50^\circ \). When \( \beta > \phi_2 \), the value of \( S_{\text{m}} \) tends to infinity.

2.2. Case of slickenside lineation with the same pitch direction as the marker traces on the fault

This is a case in which the lineation and marker trace have the same pitch direction is shown in Fig. 4. We consider two different sub-cases according to pitches of slickenside lineations and markers: situation of \( \beta > \gamma \) and situation of \( \beta < \gamma \).
2.2.1. Situation of $\beta > \gamma$

When pitch direction of the observation line (DF) is the same as that of slickenside lineation (Fig. 4a), the pitch of this line is defined as $\varphi_1$ (Fig. 4a). First, we consider that the value of $\varphi_1$ is larger than $\beta$. $\text{CG}'$ is the apparent fault displacement ($S_m$) measured along line DF (Fig. 4a). For the triangle $\text{CC}'\text{G}$, $\text{CC}' = S_t$, $\text{CG}' = S_m$, $\angle \text{GCC}' = \varphi_1 - \beta$, $\angle \text{GCC} = \beta - \gamma$, therefore by using the Law of Sines of triangle

$$S_t = \frac{S_m \sin(\varphi_1 - \beta)}{\sin(\beta - \gamma)}.$$  \hspace{1cm} (10)

Similarly, for $\varphi_1 < \beta$, we have

$$S_t = \frac{S_m \sin(\beta - \varphi_1)}{\sin(\beta - \gamma)}.$$

From these two equations, by assuming the value of $S_m$ and giving two of $\beta$, $\gamma$, and $\varphi_1$, we can calculate the tendency of the value of $S_m$.

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**Fig. 3.** Relationships between the values of $S_m$ and the values of $\varphi_1$, $\beta$, and $\gamma$, based on Eqs. (7) and (8) to the case when slickenside lineation has opposite pitch direction to marker trace on the fault. For all curves, the total displacement $S_t$ is assumed to be 100 (horizontal dash line). In (a), $\beta = 20^\circ$, $\gamma = 10^\circ$ to $90^\circ$. In (b), $\beta = 60^\circ$, $\varphi_2 = 10^\circ$ to $90^\circ$. In (c), $\varphi_2 = 50^\circ$, $\gamma = 10^\circ$ to $90^\circ$.

**Fig. 4.** Block diagrams showing that the pitches of the slickenside lineation and cutoff of the marker have the same directions. Marker 1 is parallel to marker 2. (a) The situation of $\beta > \gamma$. Lines JK and FD are two lines along which the apparent displacement are measured. (b) The situation of $\beta < \gamma$. Lines CD and BG are two lines along which the apparent displacement are measured.
with the third angle. In Fig. 5a, for \( \varphi_1 \leq \beta \), the value of \( S_m \) has a positive relationship with the value of \( \varphi_1 \), whereas, for \( \varphi_1 > \beta \), the value of \( S_m \) has a negative relationship. When the curves approaches the line of \( \varphi_1 = \beta = 50^\circ \), the value of \( S_m \) tends to infinite. This feature implies that the difference between the values of \( S_m \) and \( S_t \) become larger when the measurement line is more parallel to the cutoff of markers. In Fig. 5b, the values of \( \gamma \) has a negative relationship with the value of \( S_m \). The curves coincide each other for values \( \varphi_1 = 60^\circ \) and \( \varphi_1 = 90^\circ \), and for \( \varphi_1 = 70^\circ \) and \( \varphi_1 = 80^\circ \). The gradients of the curves for the larger values of \( \varphi_1 \) are greater. In Fig. 5c, for \( \beta < \varphi_1 \), the value of \( S_m \) has a positive relationship with the value of \( \beta \), whereas, for \( \beta > \varphi_1 \), the value of \( S_m \) has a negative relationship. The lines \( \beta = \varphi_1 \) are null lines. When the curves are close to the null lines, the value of \( S_m \) tends to show infinite value. This effect is similar to that in Fig. 5a.

Fig. 5. Maps showing relationships between the values of \( S_m \) and the values of \( \varphi_1 \), \( \beta \), and \( \gamma \). The curves are based on Eqs. (10) and (11). For the curves in (a), \( \beta = 50^\circ \), \( \gamma = 5^\circ \) to \( 45^\circ \). For the curves in (b), \( \beta = 75^\circ \), \( \varphi_1 = 60^\circ \) or \( 90^\circ \). For the curves in (c), \( \gamma = 10^\circ \), \( \varphi_1 = 30^\circ \) to \( 70^\circ \).

Fig. 6. Changes of the values of \( S_m \) with \( \varphi_2 \), \( \gamma \), and \( \beta \). For all curves, the total displacement \( S_t \) is assumed to be equal to 100 with no unit. The results are based on Eq. (14). (a) Relationship between \( S_m \) and \( \varphi_2 \), given \( \beta = 60^\circ \), \( \gamma = 10^\circ \) to \( 50^\circ \). (b) Relationship between \( S_m \) and \( \gamma \), given \( \beta = 80^\circ \), \( \varphi_2 = 10^\circ \) to \( 90^\circ \). (c) Relationship between \( S_m \) and \( \beta \), given \( \gamma = 20^\circ \), \( \varphi_2 = 10^\circ \) to \( 90^\circ \).
For Eq. (10), if \( S_m = S_n \), it can be deduced:

\[
\phi_1 = 2\beta - \gamma \tag{12}
\]

Like in the previous cases we can obtain the total true displacement not only along the slip direction but also along other special lines. On the other hand, from Eq. (11), we can infer that also in the case of \( \phi_1 = \gamma \), the measured displacement is the true displacement.

Particularly, when apparent displacement (\( S_m \)) is measured along dip, we can obtain the relationship between \( S_i \) and \( S_m \). In Fig. 4a, for the triangle CC′E, \( C'E = S_m \), \( \angle CEC = 90° - \beta \), \( \angle CCE = \beta - \gamma \), by using the Law of Sines of triangle, the relationship between \( S_i \) and \( S_m \) can be shown that

\[
S_i / \sin(90° - \beta) = S_m / \sin(\beta - \gamma) \tag{13}
\]

When pitch direction of the observation line (JK) is opposite to that of the slickenside lineation, the pitch of this line is defined as \( \phi_2 \). In Fig. 4a, \( \angle CHC = 90° - \phi_2 \), by using the Law of Sines of triangle, we obtain

\[
S_i / \sin(90° - \phi_2) = S_m / \sin(\beta - \gamma) \tag{14}
\]

For the given values of \( \beta \), \( \gamma \), and \( \phi_2 \), the tendency of the value of \( S_m \) is shown in Fig. 6. In Fig. 6a, given \( \beta = 60° \), for small value of \( \phi_2 \), the value of \( S_m \) has a negative relationship with the value of \( \phi_2 \), whereas for large value of \( \phi_2 \), the value of \( S_m \) has a positive relationship. The inflection points between the negative and positive relationship are located on the intersection between the line \( \phi_2 = 30° \) with curves. Two results can be obtained from Fig. 6b. First, for the given values of \( \beta \) and \( \phi_2 \), the value of \( S_m \) has a negative relationship with the value of \( \gamma \). Second, the curves for the larger values of \( \phi_2 \) have steeper gradients. The curves in Fig. 6c show that for the given values of \( \gamma \) and \( \phi_2 \), the value of \( S_m \) has a positive relationship with the value of \( \beta \). The gradient of curves become large when the value of \( \phi_2 \) and \( \beta \) if close to 90°.

2.2.2. Situation of \( \beta = \gamma \)

For this situation, two cases are considered (Fig. 4b). First, when the pitch direction of the observation line (DC) is the same as that of the slickenside lineation, the pitch of this line is defined as \( \phi_1 \). In Fig. 4b, \( \angle C'F \) is the fault apparent displacement (\( S_m \)) measured along cutoff DC. For the triangle CC′F, \( \angle F = S_m \), \( \angle FCC = 180° - (\phi_2 + \beta) \), \( \angle FCC = 90° - \gamma \), by using the Law of Sines of triangle, we obtain:

\[
S_i / \sin(\phi_2 + \beta) = S_m / \sin(\gamma - \beta) \tag{15}
\]

This equation is for the case of \( \phi_1 = \beta \). Likewise, for the case of \( \phi_1 < \beta \), we have:

\[
S_i = S_m \sin(\beta - \phi_1) / \sin(\gamma - \beta) \tag{16}
\]

Given values of \( \beta \) and \( \gamma \), the tendencies of \( S_m \) are shown in Fig. 7. It can be seen that for \( \phi_1 < \beta \), the value of \( S_m \) has a positive relationship with the value of \( \phi_1 \), whereas, for \( \phi_1 > \beta \), the value of \( S_m \) has a negative relationship (Fig. 7a). On the other hand, when the value of \( \phi_1 \) is close to the value of \( \beta \), the value of \( S_m \) tends to infinite. This implies that, for \( \phi_1 = \beta \) (null line), the apparent displacement cannot be obtained. Fig. 7b shows that for given values of \( \phi_1 \) and \( \beta \), the value of \( S_m \) has positive relationship with the value of \( \gamma \). The curves for smaller values of \( \phi_1 \) show larger gradients of curves. Fig. 7c presents that for \( \beta < \phi_1 \), the value of \( S_m \) has positive relationship with the value of \( \phi_1 \), whereas, for \( \beta > \phi_1 \), the value of \( S_m \) has negative relationship. Also, from the null lines, no displacement can be measured.

For Eq. (15), if \( S_m = S_i \), we can obtain

\[
\phi_1 = 2\beta - \gamma \tag{17}
\]

Again, the total true displacement can also be measured along a different line from the slip direction. In addition, in Eq. (16), we can
infer that only in the case of $\varphi_1 = \gamma$, which means in the slip direction, the measured displacement is the true displacement (i.e. $S_m = S_t$).

In addition, in Fig. 4b, for the triangle CC'E, CE' = $S_{md}$, $\angle$CC'E = 90$^\circ$ - $\gamma$, $\angle$ECC' = $\gamma$-$\beta$, then $\angle$ECC' = 90$^\circ$ + $\beta$, by using the Law of Sines of triangle, we establish the follow equations

$$S_t / \sin (90 + \beta) = S_{md} / \sin (\gamma - \beta)$$

(18)

$$S_t = \frac{S_{md} \cos \beta}{\sin (\gamma - \beta)}.$$ 

When the pitch direction of the observation line (GB') is opposite to that of the slickenside lineation, the pitch of this line is defined as $\varphi_2$ (Fig. 4b). HB' is the fault apparent displacement ($S_{ma}$) measured along the line GB'. For the triangle BB'H, BB' = $S_m$, $\angle$BB'H = $\gamma$-$\beta$, $\angle$BB'H = 180$^\circ$ - ($\varphi_2 + \gamma$), then $\angle$BB'H = $\varphi_2 + \beta$, by using the Law of Sines of triangle, we obtain:

$$S_t / \sin (\varphi_2 + \beta) = S_m / \sin (\gamma - \beta)$$

(19)

$$S_t = \frac{S_m \sin (\varphi_2 + \beta)}{\sin (\gamma - \beta)}.$$ 

For given values of $\beta$, $\gamma$, and $\varphi_2$, the changes of $S_m$ are shown in Fig. 8. From Fig. 8a, it can be seen that the value of $S_m$ has a negative relationship with the value of $\varphi_2$. Also, the curve gradient has a negative relationship with the value of $\varphi_2$. By comparing different curves, the curve gradients for smaller values of $\varphi_2$ are larger than those for larger values of $\varphi_2$. The curves in Fig. 8b indicate that the value of $S_m$ has a positive relationship with the value of $\gamma$. On the other hand, the curve gradient has a negative relationship with the value of $\gamma$. Fig. 8c shows that the value of $S_m$ has a negative relationship with the value of $\beta$. For $\varphi_2 > 45^\circ$, the curves are convex right-up, whereas for $\varphi_2 < 45^\circ$, the curves are convex left-down.

In addition, assuming the value of $S_m$ is equal to the value of $S_t$, we have:

$$\varphi_2 = \gamma - 2\beta$$

(20)

This equation shows that there is a line, different from the slickenside, along which the total true displacement could be measured.

3. Calculation of pitch of the fault slip lineation

If there are two non-parallel markers, two sets of measured data for these two markers can be obtained. Then, the pitch value ($\gamma$) of the true displacement can be estimated employing the two data sets. The attitudes of two marker horizons have many combinations that can be used to evaluate the pitch of true displacement. For example, if the two marker horizons are consistent with the prerequisite of Eq. (2), in the case of slickenside lineation with opposite pitch direction to marker traces on the fault, then the Eq. (2) can be used to calculate the value of $\gamma$. Therefore, the following relationship can be established:

$$\frac{S_{m1} \sin (\varphi_1 + \beta_1)}{\sin (\gamma + \beta_1)} = \frac{S_{m2} \sin (\varphi_1 + \beta_2)}{\sin (\gamma + \beta_2)}$$

(21)

where $S_{m1}$ and $S_{m2}$ are the apparent displacements measured along line 1 for the marker 1 and marker 2, respectively, and $\beta_1$ and $\beta_2$ are the pitch of marker 1 and marker 2, respectively.

By rearranging and simplifying, we obtain:

$$\tan \gamma = \frac{S_{m2} \sin (\varphi_1 + \beta_2) \sin (\gamma + \beta_2) - S_{m1} \sin (\varphi_1 + \beta_1)}{S_{m2} \sin (\varphi_1 + \beta_2) \cos (\gamma + \beta_2) - S_{m1} \sin (\varphi_1 + \beta_1)}$$

(22)

Other combinations between two markers can be formulated, however, to avoid similarly inferring, they are not discussed here.

4. Case study: Extensional strain in Sierra de San Miguelito, Mexico

The fault strain is due to continuous fault slip along the dip direction (Jamison, 1989; Peacock and Sanderson, 1993; Schultz and Fossen, 2002). Fault strain provides important information about fault linkage, fault formation, and lithologic influence on faulting process. A larger fault strain corresponds to a larger fault slip (Westaway and Kusznir, 1993). Therefore, the strain profiles should be similar to the displacement profiles, if the faults growth after rocks were deposited. For an isolated fault, the displacement profile has been considered with the form of cone or ellipse (Gupta and Scholz, 2000, Fig. 12c,d). Multipeak curves in the profiles suggest that the faults in the study area experienced complicated evolution and fault linkage (e.g. Segall and Pollard, 1983; Gudmundsson, 1987; Peacock, 1991; Trudgill and
Cartwright, 1994; Xu et al., 2006). Ferrill et al. (1999) reported two mechanisms of linkage: breakthrough by curved lateral propagation and breakthrough by connecting fault formation. Both linkages may cause irregular displacement distribution of fault dip along fault strike.

To apply our methods, we selected the Sierra de San Miguelito as a detailed study area (Fig. 9). The study area is within an elevated plateau in central Mexico located in the south of the Mesa Central. The fault system in San Miguelito has been regarded as having a "domino style" because it consists of sub-parallel faults that systematically tilt the volcanic beds to the NE (Labarthe-Hernández and Jiménez-López, 1992; Nieto-Samaniego et al., 1997, Xu et al., 2004b). On the whole the individual faults have a strike direction of N20°W–S60°E and dips to the SW. The fault slickenside lineations are observed to plunge SW (occasionally SE) with a pitch of 75°–85°. There are some faults with N–S or NE-SW strike direction. Their main activity was dated by Nieto-Samaniego et al. (1999) between 30.0 and 26.8 Ma, coeval with the emission of the Cantera Ignimbrite and ended with the emplacement of the upper Panalillo Rhyolite (Labarthe-Hernández and Jiménez-López, 1992).

In the studied area, the Cenozoic units are: Portezuelo Latite, San Miguelito Rhyolite, Cantera Ignimbrite, Panalillo Rhyolite, and Halcónes Conglomerate. A unwelded member of the Cantera Ignimbrite is selected as a marker for measurement. Xu et al. (2004b) documented a vertical shear mechanism of faulting and bed tilting. For vertical shear, the extension produced by any planar normal fault is equal to its heave. Thus, the horizontal distance between the footwall cutoff of one fault and the hanging-wall cutoff in the next fault will remain constant as deformation proceeds, maintaining its initial value ($L_o$ in Fig. 10). So, we have $DC=BC′=L_o$ and the heave is $h=D′B=\cot(\alpha)\sin(\theta)$. Length before deformation is

$$L_o = L_0 \cos(\theta).$$

(23)

Therefore, the extension is

$$e = \frac{h}{L_o} = \cot(\alpha) \tan(\theta).$$

(24)

Whether the extension calculated from this equation is true depending on the orientation of the slickenside lineation and bed azimuth. For the faults with pure dip-slip ($\gamma=90°$) and the bed with strike parallel to the fault strike ($\beta=0°$), the calculated extension is true. For the oblique faults ($\gamma\neq90°$), and the bed with strike not parallel to the fault strike ($\beta\neq0°$), a correction factor should be added for calculation of true extension. For example, if the fault slip and

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**Fig. 9.** Geological map of the Sierra de San Miguelito.

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**Fig. 10.** Sketch to illustrate the relationships among parameters for domino faults assuming vertical shear supposed by Westaway and Kusznir (1993).
marker cutoff are consistent with Eq. (2), thus the equation to obtain the true extension in an oblique fault will be

\[ e = \frac{\sin(\gamma + \beta)}{\sin(\varphi_1 + \beta)} \cot(\alpha) \tan(\theta) \]  

(25)

where \( \frac{\sin(\gamma + \beta)}{\sin(\varphi_1 + \beta)} \) is a correction factor.

If the apparent displacement is measured along fault dip, the value of \( \varphi_1 \) is 90°, then, Eq. (25) will be

\[ e = \frac{\sin(\gamma + \beta)}{\cos(\beta)} \cot(\alpha) \tan(\theta) \]  

(26)

In order to avoid confusion in terminology, now we will use the term “fault strain” for the extension calculated as explained above. The calculated results from Sierra de San Miguelito are shown in Figs. 11 and 12, where the horizontal coordinate is normalized distance (distance/length). The characteristics of the fault strain profiles show very irregular and multipeak curves for the faults that are intersected or overlapped by other faults (Fig. 11). The largest fault strain reaches ca. 50% along fault 1. The smallest fault strain is ca. 8% along faults 14 and 16. For isolated faults shown in Fig. 12, more regular profiles were obtained, but they are not very consistent with the displacement profiles of other published isolated faults (e.g. Dawers et al., 1993; Dawers and Anders, 1995; Fossen and Hesthammer, 1997). The correction factors from the section AA’ are shown in Table 2. It can be seen that the correction factors can be either larger or smaller than 1. The largest value is 1.15, and the smallest value is 0.97. These results imply that the differences between the true extensions and apparent

![Fig. 11. Fault strain profiles along the strike of faults with overlapping or intersecting geometries in the Sierra de San Miguelito, Mexico. These profiles are more irregular than those of isolated faults in Fig. 12. Distance is measured from the northwestern end of each fault.](image-url)
extensions are not very large. This is because the values of $\gamma$ do not deviate much from 90° ($72°$–$78°$).

5. Application limitations

When a fault is reactivated (e.g., Celerier, 1988; Alaniz-Alvarez et al., 1998), the observed slickenside lineation may be due to the last movement of the fault. In this case, the calculated total displacement will not represent the amount of total fault movement. This effect is shown in Fig. 13. Two slip generations of the fault (CC$^*$ and C$^*C^*$) are assumed. If only last slickenside lineation (C$^*C^*$) is observed, the obtained displacement according to Eq. (5) is FC$^*$. This value is neither equal to total displacement (CC$^*$=S$^*$) nor the last displacement C$^*C^*$.

![Fig. 12. Fault strain profiles along the strike of isolated faults in the Sierra de San Miguelito. These profiles are more regular than those of faults with overlapping or intersecting geometries in Fig. 11, but still deviate from the theoretical displacement profile. Distance is measured from the northwestern end of each fault. In (e) and (f), the ideal displacement profiles are present according to Gupta and Scholz (2000).](image)

![Table 2](table)

<table>
<thead>
<tr>
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<th>6</th>
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<tr>
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<td>210/64</td>
<td>250/45</td>
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<td>215/55</td>
<td>238/58</td>
<td>226/75</td>
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<tr>
<td>A2/$\theta$ ($^\circ$)</td>
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<td>30/23</td>
<td>20/25</td>
<td>31/28</td>
<td>42/30</td>
<td>48/24</td>
<td>25/22</td>
<td>30/15</td>
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<td>76NW</td>
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<td>76SE</td>
<td>75SE</td>
<td>76SE</td>
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</tr>
<tr>
<td>$\beta$ ($^\circ$)</td>
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<td>17.7</td>
<td>5.5</td>
<td>36.3</td>
<td>14.5</td>
<td>10</td>
<td>15.8</td>
<td>4.4</td>
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<td>5.8</td>
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<tr>
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<td>0.99</td>
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<td>0.99</td>
<td>1.01</td>
<td>0.99</td>
<td>0.97</td>
</tr>
</tbody>
</table>

A1 is the dip direction of the fault. A2 is the dip direction of the bed. A3 is the pitch direction of slickenside lineation on fault. $\alpha$ is fault angle. $\gamma$ is pitch of slickenside lineation. $\beta$ is pitch of cutoff. $\xi$ is correction factor.
Sometimes, normal faults show opening or pull-apart structures (e.g. Acocella et al., 2003; Gasperini et al., 2003). In this case, the total displacement cannot be calculated, because the cutoffs of the same marker cannot be observed on one fault surface (Fig. 14). For example, in Fig. 14b, if the fault has opening A'A'' or B'B'', the amount of the fault movement is CC''. However, the total displacement on fault is CC'. The line CC'' is not on the fault surface. Evidently, the amount of CC'' is not equal to CC'. To calculate CC' in this case, the value of C'C', CC'', and \(<\)C'CC'' are needed. However, the value of \(<\)C'CC'' is difficult to obtain in practice.

Our methods assume that faults are planar and have uniform slip across the portion of the fault surface analyzed. These conditions are not met in some cases.

For the scissor faults, the slip is not uniform along both the strike and dip in the ranges of analysis (Fig. 15a). Also the slickenside lineations, commonly, are not a straight line. This produces the change of the value of \(\beta\) across the portion of analysis. For the listric faults, the fault dip varies with depth (Fig. 15b). As a result, the value of \(\beta\) will be different for the same marker. Thus, the established equations cannot be used to calculate the true displacement. Finally, if the beds along the fault zone experienced ductile deformation, the calculated true displacement will have error. In Fig. 15c, the true displacement is AB for the bottom of bed b. The calculated true displacement is AC. The error of the true displacement is BC.

In general, it is dependent upon the scale of analysis to fulfill all requirements of the geometric models. If the fault at the scale of observation is plane, the slickenside lineations are straight, and the host rock is considered as rigid body, then the models can be well applied. This is similar to the other analysis of deformation for which it is necessary to consider that the obtained results are valid only at the scales where the deformation are analyzed. For example, the deformation is heterogeneous or homogeneous, depending on the scale of observation.

![Fig. 13. Diagram showing two movements of a fault. The trajectory of fault movement is C→C″→C'. Total true displacement is S_t=CC″. The last displacement is C'C″, which is not equal to the total true displacement (CC″).](image)

![Fig. 14. (a) Diagram showing a fault with no opening perpendicular to the fault strike. The total displacement on fault is S_t=CC″. (b) If the same fault in (a) has a further opening A'A″ or B'B″, the directly measured offset for the marker is CC″. This value is not equal to CC. In this case, the value of S_t (CC″) cannot be directly measured.](image)

![Fig. 15. (a) On a scissor fault, the magnitude and direction of fault are changeable. (b) For a listric fault, the fault dip along the dip is different. (c) For a ductile fault, the fault displacement is not the true slip due to ductile deformation.](image)
6. Conclusions

The true and apparent displacements of a fault are related in a geometrical way. When the fault apparent displacement from only one marker can be measured and slickenside lineation on the fault can be observed, we established a series of equations for determining the relationship between the true displacement and apparent displacement according to the different combinations among pitch directions of slickenside lineation, cutoffs of markers and observation lines. Three results are obtained from this scenario. First, if the apparent displacement is taken as the true displacement, the latter may be either overestimated or underestimated. Second, for given values of $\beta$ and $\gamma$, the measured displacement $S_m$ (apparent displacement) is dependent on the pitch of the observation line ($\varphi$). When the observation line is parallel to the cutoffs ($\varphi=\beta$), the displacement cannot be observed. The value of $S_m$ is either positively or negatively related to the pitch of observation line ($\varphi$), the pitch of cutoffs ($\beta$), and the pitch of slickenside lineation ($\gamma$). Third, the total true displacement can be obtained along one particular line except for the slip direction on the fault plane. The pitch of the particular lines can be calculated by the values of $\gamma$ and $\beta$. On the other hand, if the fault apparent displacements from two non-parallel markers can be measured, we inferred some equations to determine the direction of fault slip.

The methods are applied to calculate the extension due to the normal faults of San Miguelito in Mesa Central, Mexico. The results indicate that the largest fault strain reaches ca. 0.508 and the smallest fault strain is ca. 0.08. For the isolated faults, the strain profiles along the fault strikes are similar to the displacement profiles and show the largest strain near to center. For the faults with overlapping or intersecting geometries, the strain profiles show multipeaks and more irregular than for the isolated faults.

Acknowledgements

This work was supported by the research projects D.01003 of the Instituto Mexicano del Petróleo, and 89867 and 049049 of Conacyt. We wish to thank A. Billi and an anonymous reviewer for their pertinent comments.

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