Three-dimensional displacement fields measured in a deforming granular-media surface by combined fringe projection and speckle photography

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Abstract
3D displacement fields on a diffuse surface are measured by a combination of two optical methods, fringe projection and speckle photography. The use of a single camera for recording information from the two methods implies that no calibration procedures are necessary, as is the case in stereoscopy-based techniques. Out-of-plane displacements are measured by fringe projection whereas speckle photography yields the 2D in-plane component. By using this technique, we analyze in detail the morphological spatial–temporal evolution of an analogue model of the Earth’s crust while subjected to compression forces. We discuss the experimental results and their relevance to the micromechanics of a surface of dry, non-cohesive and dilatant granular media. The results show that the combination of fringe projection and speckle photography is well suited for this type of study and allows the characterization of strain at the grain scale.

Keywords: fringe projection, 3D deformation, speckle photography, analog models, geological deformation, strain

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The non-contact measurement of full displacement fields for a body subjected to deformation is of great interest in several fields of engineering and geosciences. In general, a complete characterization of 3D displacement fields must contain the 2D in-plane and the out-of-plane displacement components. If the target deformational event is relatively slow then all three components of displacement may be obtained simultaneously. Several optical techniques have been proposed in order to accomplish such a task [1–18]. For instance, if all the displacement components are of the order of a few micrometers then either electronic speckle pattern interferometry (ESPI) [1, 2], digital holography [3, 4] or Moiré interferometry [5] may be used. In the case of an in-plane component much larger than the out-of-plane displacement, a combination of digital speckle photography and ESPI is suitable [6]. Speckle photography is a widely used technique for measuring in-plane displacements in solids [7–10] and in fluid flows [11, 12]. In speckle photography, speckle patterns before and after deformation of a specimen are captured by a CCD camera [13]. Speckle patterns are generally produced by light scattered from an optically rough surface when illuminated by coherent light [14]. The two resulting speckle images are divided into subimages of generally
32 × 32 pixels and for each subimage an average displacement vector is computed via digital cross-correlation. Fast Fourier transform is an efficient algorithm for carrying out this task. In speckle photography, decorrelation (change of the structure) of the speckle patterns is the main limiting factor for the measuring range; however, displacements over tens of mean speckle diameters may be measured [13]. Three-dimensional displacement fields can be retrieved from speckle photography by using two cameras [7, 8, 15, 16], but calibration procedures are necessary since alignment and perspective correction between images from both cameras cannot be avoided [7, 17]. On the other hand, fringe projection is a technique widely used for obtaining the 3D contours of objects [18–21], which consists of taking a photograph of the object under investigation when binary fringes are projected onto it. The projected grating leads naturally to the use of fringe carrier methods for optical phase assessment [21]. Recently, it has been shown that fringe projection can be applied for the measurement of out-of-plane deformation [22, 23]. In this case, the object is imaged at two different deformation states and the difference in the corresponding optical phases defines the relative deformation between both states [24]. The range of measurement of fringe projection goes from a few tens of micrometers to millimeters. Thus, such a range complements that obtained with interferometric and speckle methods.

In order to analyze 3D displacements of the order of a fraction of one millimeter, overcoming the above-mentioned problems for stereoscopy-based methods, recently the combination of speckle photography and fringe projection was proposed [23, 25]. In this case, a single CCD camera records, in one image, a speckle pattern and the resulting fringe pattern projected on a deforming surface during successive strain increments. In this work, we use this technique for the analysis of geological models, but in this case, to increase the precision of the results [25], two separate images are used, one for capturing a speckle pattern and one for capturing a modulated fringe pattern.

The separation in time between consecutive images is limited by both the blanking time of the CCD camera, which may be smaller than 1 µs, and by the response time of a beam shutter, which for a mechanical device is of the order of milliseconds.

The suitability of the proposed technique for relatively slow events is tested in a series of physical experiments performed in a sandbox simulating deformation of the Earth’s surface during shortening. Shortening was produced by compressing an originally uniform-thickness and horizontally layered mechanical stratigraphy composed of quartz sand and silicone layers. Since the out-of-plane component is larger than the in-plane displacement, the use of fringe projection in conjunction with speckle photography is well suited for this type of analysis. Finally, the displacement field obtained by optical techniques is compared with those obtained with traditional image analysis and some implications for the mechanics of granular media are discussed.

2. Theory of optical techniques

2.1. Digital speckle photography

When an optically rough specimen is illuminated by coherent light a speckle pattern is formed at the plane of the sensor of a CCD. As deformation is applied to the specimen the speckles are correspondingly displaced [26]. The relative displacement of the speckles for two consecutive deformation states is obtained by cross-correlating subimages corresponding to images of the two different states [9]. A schema showing the optical set-up for combined speckle photography and for fringe projection is shown in figure 1.

Let \( I_1(x, y) \) and \( I_2(x, y) \) be the intensity distributions of the speckle patterns at a point \((x, y)\) at the object plane for a reference and for a deformed state, respectively. Denoting by \( u(x, y) \) and \( v(x, y) \) the relative speckle displacements in the \(x\) and \(y\) directions for those two states, then it is possible to write

\[
I_2(x, y) = I_1 \left[ x - u(x, y), y - v(x, y) \right].
\]

By denoting the Fourier transform of \( I_1(x, y) \) and \( I_2(x, y) \) by \( F_1(f_x, f_y) \) and \( F_2(f_x, f_y) \), then the power spectrum distribution function can be calculated in the Fourier domain \((f_x, f_y)\) as [9, 12]

\[
P(f_x, f_y) = \frac{F_1(f_x, f_y)F_2^*(f_x, f_y)}{|F_1(f_x, f_y)F_2(f_x, f_y)|} = \exp \left[ i \left( \phi_1(f_x, f_y) - \phi_2(f_x, f_y) \right) \right],
\]

where \( \phi_1(f_x, f_y) \) and \( \phi_2(f_x, f_y) \) are the corresponding spectral phases of \( F_1(f_x, f_y) \) and \( F_2(f_x, f_y) \), respectively, and \( i = \sqrt{-1} \). In equation (2) the * denotes the complex conjugate operation. By the translation property of the Fourier transform

\[
\phi_1(f_x, f_y) - \phi_2(f_x, f_y) = 2\pi \left( u(x, y)f_x + v(x, y)f_y \right).
\]
2.2. Fringe projection

Binary fringes of period $p$ are projected onto the surface of the specimen, in our case by means of a computer projector. For a flat surface, and when the illumination distance is sufficiently large, the period $p$ can be taken as a constant throughout the specimen as shown in the optical set-up schema of figure 1 for fringe projection. In this case the projected pattern can be described by

$$f(x) = \sum_{n=0}^{\infty} c_n \cos \left( \frac{2\pi n x}{p} \right),$$  \hspace{1cm} \text{(4)}

where $c_n$ are the coefficients of the Fourier series and $n$ is an integer. However, as shown below, when a bandpass filter in the frequency domain is applied to (4) only the first two terms of the series are sufficient for the analysis. Then, the projected grating can be expressed as

$$f(x, y) = a(x, y) + b(x, y) \cos \left( \frac{2\pi}{p} x \right),$$  \hspace{1cm} \text{(5)}

where, by comparison with interferometry, $a(x, y)$ is known as the background illumination and $b(x, y)$ is the contrast of the fringes. As the CCD camera records this pattern, slight perspective effects arise at the edges of the region of observation (this may be avoided by the use of a telecentric lens) and the recorded period varies with position $(x, y)$. Then the image recorded by the CCD can be expressed as

$$I(x, y) = a(x, y) + b(x, y) \cos \left( \frac{2\pi}{p} x \right),$$  \hspace{1cm} \text{(6)}

where a constant factor taking into account the conversion from analog format in volts to digital form in gray levels has been omitted. Equation (6) can be written as [27]

$$I(x, y) = a(x, y) + b(x, y) \cos \left( \frac{2\pi}{p} x + \theta(x, y) \right),$$  \hspace{1cm} \text{(7)}

where $\theta(x, y)$ is the modulation term related to perspective and optical aberration effects.

When the specimen is subjected to deformation an additional variation of the period is produced. Then

$$I(x, y) = a(x, y) + b(x, y) \cos \left( \frac{2\pi}{p} x + \theta(x, y) + \phi(x, y) \right),$$  \hspace{1cm} \text{(8)}

where $\phi(x, y)$ is the phase modulation introduced by the deformation and is related to the resulting out-of-plane displacement $w(x, y)$ by [22, 27]

$$w(x, y) = \frac{\phi(x, y)}{2\pi} \frac{p}{\tan \alpha},$$  \hspace{1cm} \text{(9)}

where $\alpha$ is the mean angle between the observation and illumination directions. Equation (8) implies that, once $\phi(x, y)$ is known, the corresponding out-of-plane displacements can then be measured. One way to obtain $\phi(x, y)$ is by taking the difference of the arguments of equations (8) and (7):

$$\left[ \frac{2\pi}{p} x + \theta(x, y) + \phi(x, y) \right] - \left[ \frac{2\pi}{p} x + \theta(x, y) \right].$$

It can be noted that in the latter operation perspective and aberration effects are canceled. To calculate the arguments of equations (8) and (9), the Fourier method may be applied [22]. For instance, to compute the argument of equation (8), first we rewrite equation (8) as

$$I(x, y) = a(x, y) + \frac{1}{2} b(x, y) \exp \left[ i g(x, y) \right] \exp \left( \frac{i 2\pi}{p} x \right) + \frac{1}{2} b^*(x, y) \exp \left[ -i g(x, y) \right] \exp \left( -\frac{i 2\pi}{p} x \right),$$  \hspace{1cm} \text{(10)}

where $g(x, y) = \theta(x, y) + \phi(x, y)$. To obtain $g(x, y)$, the Fourier transform of equation (10) is taken:

$$I_P(f_x, f_y) = A(f_x, f_y) + B(f_x - f_0, f_y) + B^*(-f_x - f_0, f_y),$$  \hspace{1cm} \text{(11)}

where $f_0 = 1/p$, $A(f_x, f_y) = \Im \{a(x, y)\}$ and $B(f_x, f_y) = \Im \{\frac{1}{2} b(x, y) \exp [i g(x, y)]\}$, with $\Im \{\}$ denoting the Fourier transform operator. By applying a bandpass filter to the spectrum produced by equation (11), we can obtain

$$I_P(f_x, f_y) = B(f_x - f_0, f_y).$$  \hspace{1cm} \text{(12)}

In this step, any higher harmonic component of the series given by equation (4) is removed. Next, we take the inverse Fourier transform of equation (12):

$$\mathcal{F}^{-1} \left\{ B(f_x - f_0, f_y) \right\} = \mathcal{F}^{-1} \left\{ \Im \left\{ \frac{1}{2} b(x, y) \exp [i g(x, y)] \right\} \right\}_{f_x - f_0} \rightarrow \frac{1}{2} b(x, y) \exp [i g(x, y)] = R(x, y) + i M(x, y),$$  \hspace{1cm} \text{(13)}

where

$$R(x, y) = \frac{1}{2} b(x, y) \cos \left[ 2\pi f_0 x + g(x, y) \right]$$  \hspace{1cm} \text{(14)}

and

$$M(x, y) = \frac{1}{2} b(x, y) \sin \left[ 2\pi f_0 x + g(x, y) \right].$$  \hspace{1cm} \text{(15)}

From equations (14) and (15), the desired phase term can be found from

$$2\pi f_0 x + g(x, y) = \frac{2\pi}{p} x + \theta(x, y) + \phi(x, y) \Rightarrow R(x, y) \rightarrow \tan^{-1} \left( \frac{M(x, y)}{R(x, y)} \right).$$  \hspace{1cm} \text{(16)}

Once the difference of the arguments is calculated, the resulting phase is proportional to the out-of-plane displacement via equation (9). This phase term may present the common $2\pi$-phase ambiguity problem [27], but it may be remedied by the use of any phase unwrapping algorithm [28]. Additional problems arise when the displacements become greater than a few grating periods [21], so it is convenient to adjust the temporal sampling of the event in such a way that the total displacement becomes the sum of incremental displacements. For this, an updated reference image may be used [29].
3. Experimental set-up

3.1. Analog modeling of deformation and displacement field obtained by photography analysis

The proposed combination of techniques was tested in a geosciences application. Our experiments simulate deformation of the Earth’s surface when a mechanically layered profile is subjected to compressive forces. A detailed description of the natural prototype reproduced by our experiments is beyond the scope of this paper and will be presented elsewhere; here, we describe briefly the main characteristics of the model and the necessary considerations to validate the optical methods. A schema showing the design of the model is presented in figure 2. The model was constructed within a Plexiglas box with an advancing vertical wall driven by a step motor. The Plexiglas box and motor were placed above a pneumatic table in order to reduce as much as possible interference due to motor vibration. Horizontal layers of sand and silicone were successively added to the box. Sand was sieved and poured into the box; the excess of sand was carefully scraped until a horizontal surface was obtained. The silicone layer was frozen and cut to fit the dimensions of the box. Above the silicone layer, white and colored quartz sand was added successively in layers. Thus, initial conditions of the three-layered model included layers of brittle behavior (basal and upper sand layers) of 1 and 2 cm, respectively (density of 1300 kg m\(^{-3}\)), separated by an intermediate viscous layer of 0.5 cm. Dry and rounded quartz sand has a brittle behavior following a Mohr–Coulomb criterion of faulting with an internal friction coefficient of 0.6 [30]. The viscous layer was constructed with PDMS silicone fluid with a density of 1195 kg m\(^{-3}\), dynamic viscosity of \(3 \times 10^4\) Pa s for the experimental strain rate, and presents a near-Newtonian flow curve. The model is a generic representation of mountain forming processes in naturally shortened areas with an involved decollement layer (the viscous fluid). Although it does not represent a specific area, we consider a geometrical scale in which 1 cm represents approximately 1 km of the Earth’s crust. The advancing wall displaced at a velocity of 2.2 cm h\(^{-1}\) until bulk shortening (b.s.) reached around 20%.

The surface of the model is composed of granular, non-cohesive and dilatant material with a homogeneous size of around 250 μm and a loose fabric (zoomed area of figure 2(a)). When the advancing wall moves deformation is controlled by frictional sliding at the mechanical contact between particles and accommodated by their displacement and rotation. The advancing wall’s first effect is to produce a compacted fabric and ultimately granular flow. In natural geological settings the lateral deformation is thought to be accommodated mainly by layer parallel shortening, folding and imbricate thrusting.
In the models, the elastic component and inertial forces are expected to be negligible during the experiments [33, 34], and parallel layer shortening is thought to be the result of void lost between particulate elements. As deformation advances the loading produces plastic strain hardening until preferential accommodation of grains produces failure along shear zones propagating from the base of the sand of the model to the surface and facilitating vertical displacement of sand (in our case folding or thrusting), and thickening of the model. It has also been suggested that failure (formation of shear zones) in sand is associated with the maximum rate of dilation [33–35].

At the length scale generally used to interpret deformation in photographs of analog modeling the surface can be reasonably considered as a continuum. In this case a square element of a grid drawn with colored sand above the surface of the models can be considered a representative area in which the deformation is an average of the individual displacements of the surface constituents. In this way, the displacement field on the sand surface of the model (see figures 4 and 5) can be obtained directly by following passive markers through deformation in top-view photographs (see [36] for details on the technique). We made such an analysis in order to have a reference for the results obtained by optical methods. Crossings between vertical and horizontal lines of the colored sand grid are selected as passive markers and their positional change can be followed between successive top-view photographs taken during deformation. In this way we can obtain a picture of the in-plane displacements but not the out-of-plane deformation, which can be obtained by other methods such as laser scanning [36]. The resolution achieved with this method depends on the size of the grid drawn above the model, on the precision with which markers can be followed, and on the time interval of photograph recording. A major disadvantage is that, with the advance of deformation, some passive markers become buried below advancing material or sand collapses.

3.2. Optical set-up

For the recording of images, a PCO CCD camera of 1392 × 1024 pixels was used. The settings of the camera lens were f# of 4 and focal length of 12 mm. The exposure time for the camera was set to 40 ms. Subimages of 21 × 21 pixels were used for the speckle photography results. Images were captured every 3 min successively for both speckle photography and fringe projection and the time between the capturing of a fringe pattern and a speckle pattern was 1 s, which is sufficiently short to consider simultaneity of both measurements, considering that the model is deformed relatively slowly. The light source for speckle photography was a 30 mW He:Ne laser and for fringe projection a Dell projector. For fringe projection, a period of the grating at the surface of the model of 3.025 mm was selected and the mean angle of illumination was 20°. For maximum sensitivity of the set-up, the binary fringes were projected parallel to the moving wall. The displacement components for different times after the onset of deformation were calculated by the use of an updated reference [29]. In this case, incremental deformations between consecutive measurements separated by 3 min are first obtained and then summed.

4. Results and discussion

In order to facilitate the description of the experimental results, in the following top-view figures the short side of the models is aligned vertically (y direction) and both deformation and moving wall advance toward the right side or foreland (x direction). Two experiments with subtle differences in initial conditions were monitored with optical techniques. We discuss the results of one of the experiments and compare them with an identical initial condition experiment interpreted from top-view photographs. The experiments show the evolution of a thrust wedge above a basal decollement with a predominant front vergence exemplified in several natural examples, for example those listed in [37]. The resulting deformation features are similar to previous experiments (see, for example, [38] and references therein for a recent set of similar experiments). Strain observed in the surface is characterized by the progressive uplift and advance of the thrust wedge controlled internally by thrusting and folding (figure 2(b)). The wedge front is slightly deflected toward the advancing wall near the lateral Plexiglas walls because of boundary effects caused by friction with the model walls. Examples of raw images recorded for fringe projection and speckle photography, at an advanced deformation stage, are shown in figure 3. In both cases the strain of the surface and the trace of the deformation front are readily observed. For speckle photography, due to the low power of the laser used, some small areas of the observation region could not be illuminated appropriately so in those regions the in-plane information was not considered in the results.

4.1. Displacement fields obtained by optical techniques and top-view photographs

The analysis of the full field of vision resulted in distinctive displacement fields yielding valuable and detailed information of trajectories of displacements and strain of the originally flat surface of sand. The temporal evolution of the 3D displacement for the geological models for the case analyzed is shown in the left column of figure 4. Comparison with the incremental displacement measured in top-view photographs (right column in figure 4) is discussed below. On top of each of the plots, the maximum values of the out-of-plane and of the in-plane displacement vector and its orientation are indicated. This last vector is also highlighted by a thicker arrow in the plot. The images were divided into 50 × 36 subimages. For displaying purposes, only 25 × 18 displacement vectors are shown. The results shown in figure 4 (left column) correspond to different times after the onset of deformation. For each of these times, the corresponding displacement components are the summation of relative displacements measured between two consecutive measurements and separated by intervals of 3 min. The percentage of bulk shortening for each figure is calculated by \((L_f - L_i)100/L_i\), where \(L_i\) is the initial horizontal length of the model and \(L_f\) is the deformed length.
4.1.1. Strain of the surface and heterogeneity of particle kinematics. The results of this work are intended to be used to perform a characterization of the mechanical evolution of thrust wedges in analog models of geological deformation. Geological deformation is characterized by extremely slow velocities (mm/year) and large volumes (millions of km³), in which viscous forces dominate over inertial forces (i.e. small Reynolds numbers) and the assumption of steady flow or steady deformation history conditions is a valid first-order approximation when accurate deformation trajectories are unknown. However, recent studies suggest that most natural deformations need to be treated as heterogeneous and non-steady [39]. The analog modeling technique provides a simple way to study in the laboratory the natural deformation processes, with material such as sand that is thought to behave mechanically similar to large-scale mixtures so as to reproduce realistically the natural processes of deformation [40–42]. With the development of precise and high-resolution characterization techniques, such as presented in this paper, the study of displacement vectors during deformation of the analog models increases the possibilities of success in interpretation of natural geological structures and their trajectories of deformation.

The full displacement field in the surface of the model is the sum of the in-plane and out-of-plane displacements components: $u(x, y)$ and $v(x, y)$, and $w(x, y)$, respectively; the deformation field can be fully described by displacement components of pure translation, distortion or stretching, and rotation. We illustrate these components in figures 4(d)–(f) and 5. The sequence shown in figures 4(d)–(f) shows the incremental deformation trajectories of selected passive markers in the surface of a model. Gray and black discontinuous lines in figure 5(b) represent undeformed and deformed states, respectively, of the sand grid drawn above the area photographed in figure 5(a). As mentioned before, in this type of analysis the out-of-plane displacements cannot be measured. The translating component is shown in figure 5(b) by the displacement of lines along the $x$ direction without rotation (i), whereas (ii) and (iii) indicates counterclockwise and clockwise rotation, respectively, of the marker lines oriented initially along the $x$ direction. An increase in area is noted in (iv) by the change of length of the line between markers, and in (v) there is a decrease in area accommodated by thrusting, whose trace on the surface is shown by a discontinuous line in (vi). Note that the division between contiguous square markers has disappeared below thrusting.

In the finite deformation analysis performed on the surface of the model by optical methods we are mainly concerned with the instantaneous rotating and stretching components. To this end further relevant information can be obtained from the displacement vector maps [43]. A measure of displacement field rotation is provided by the out-of-plane vorticity defined as the local angular rotation of the in-plane displacements around an axis perpendicular to the plane, $\omega = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial y}$, where $u$ and $v$ are the in-plane displacement components along the $x$ and $y$ directions, respectively. A negative value of vorticity indicates a clockwise rotation.

Shear strain resolved in-plane is given by $\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ and it is related to the deformation caused by conjugate pairs of shear stress. An additional derived quantity is the extensional out-of-plane or normal strain $\varepsilon_{zz} = \frac{\partial w}{\partial z}$, for which a first-order approximation is estimated by assuming incompressibility of the medium as $\varepsilon_{zz} = -(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$. This parameter can be described as the net flow across the boundaries of a given contour. In the present analysis, the contour corresponds to the integration path used for calculation of the vorticity, which corresponded to a rectangular grid of $3 \times 3$ velocity vector cells, or equivalently a rectangle of $63 \times 63$ pixels.

Considering that the greatest strain is normal to the plane, then $\varepsilon_{zz}$ may be taken as the principal strain $\varepsilon_1$ and $\varepsilon_2 + \varepsilon_3$ are contained in the same plane as the shear strain $\varepsilon_{xy}$. Then a suitable approximation of the rate of volumetric strain ($\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$) during consecutive deformation steps may be calculated as $\varepsilon_v = \varepsilon_{zz} + \varepsilon_{xy}$.

From figures 4 and 5 we notice three main zones of deformation: one between the moving wall and the sloped part, the sloped zone, and the foreland. In figure 4(a) the larger in-plane displacement of 1.13 mm is located near the advancing wall and the major changes in direction are associated with the larger out-of-plane displacements in the sloped part. A major decrease of in-plane displacements is delineated by the leading edge of deformation. However, minor 3D displacements are observed to be widespread in the foreland area. The heterogeneity of strain defined by the displacement field is readily noticed from figures 4(b)–(d). Major rotations in the displacement field are evidently caused by interaction of the model with the lateral Plexiglass walls.

![Figure 3. Raw images for (a) fringe projection and (b) speckle photography, at 1.5 h after the onset of deformation. The leading edge (front) of deformation can be noted in both figures.](Image)
Figure 4. Temporal evolution of the 3D displacement of the geological model measured by optical techniques (left column) and comparison with displacements observed by top-view photographs for two experiments with a similar experimental set-up. For the optical results, the out-of-plane displacement is indicated by color levels and the in-plane by arrows. Maximum values of out-of-plane (Max op) and in-plane (Max ip) displacements as well as the direction vector of the Max ip are indicated for each deformation stage. Differential 3D displacements are measured for successive images every 3 min and added at times of: (a) 24 min, (b) 60 min and (c) 96 min after the onset of deformation. For the photograph results the incremental trajectory of selected passive markers (blue solid lines) is shown for: (d) 1.3%, (e) 8% and (f) 15.6% of bulk shortening, respectively. Red dashed lines represent trajectories that cannot be followed further or were lost in previous deformation steps. In both cases the leading edge of deformation is indicated by a discontinuous line. Here, SP stands for speckle photography and FP for fringe projection.

As an example of the detail obtained with the combined SP and FP techniques, vorticity, shear strain and normal strain were computed from the displacement field obtained for a bulk shortening of around 3.7% (figure 4(a)) and the results are presented in figure 6. On top of each image the maximum and minimum values for the corresponding quantities are included. The image was selected for an early stage of deformation with a limited out-of-plane deformation caused mainly by parallel layer shortening and folding in the brittle sand layers and thickening of the viscous layer; thrusting had a limited
Figure 5. (a) Enlargement of an area of figure 4(e) showing the top-view photographs; (b) summary of features interpreted from the deformation between figures 4(d) and (e) for the selected area: (i) translation, (ii) counterclockwise rotation (related to positive vorticity), (iii) clockwise rotation (related to negative vorticity), (iv) increase of area (related to a combination of in-plane and out-of-plane strain; shown by the arrowed gray line and the final black line), (v) decrease of area caused by (vi) shearing, in this case shear is resolved by thrusting (discontinuous line) of the left block above the right block (the triangles indicate the uplifting block), and left-lateral shearing along the trace of thrusting.

Figure 6. Derived quantities are computed from the instantaneous three-dimensional displacement field of figure 4(a); (a) 3D displacement (the units of the color scale are mm), (b) out-of-plane vorticity, (c) in-plane shearing strain and (d) normal extensional strain. The in-plane displacement is shown superimposed on the derived quantities. In this case, and unlike figure 4, no subsampling of displacement vectors is done, i.e. $50 \times 36$ vectors are included.

Influence on surface deformation. The combined displacement fields are presented in figure 6(a). High values of vorticity can be related to incipient dislocations caused by the lateral differences in the advance of deformation transmitted from friction at the boundaries. Correlation of these high values of vorticity and the alignment of highs or lows in shear strain (see figure 4(c)) suggest the presence of preferential channels of particle migration during surface shearing. This can explain the advance of the leading edge of the deformation trace in the upper part of figure 4 associated with a homogeneous local displacement field oriented towards the NE. A maximum in normal strain is also present in this zone (figure 6(d)).

A profile of the displacement components yields more information of the area where the trace of the leading edge of deformation is less advanced (figure 7). The geometry of the deformation zones defined before can be identified more readily by tracing a side view of the out-of-plane displacement component ($w$ in figure 7). A high front slope of the relief characterizes the profile. For display purposes the in-plane displacement components ($u$ and $v$) were enlarged.
three times in the vertical scale (shown on the right side of the figure). On the left side of the deformation front both in-plane displacement curves have a sinuate form with coinciding highs and lows decreasing toward the foreland and correlating with maximum and minimum values of in-plane strain (figure 6(d)). Fluctuation between positive and negative values may be indicative of flow instabilities (such as vortex-like structures) related to small differential displacements of individual grains. Negative values of \( u \) indicate displacement toward the advancing wall. An interesting zone related to the advance of deformation in the front of the leading edge is noted to the right of the \(-1\) cm position in the profile. In this area, positive values of \( u \) suggest differential compaction with peaks at 1 and 5 cm. The compaction of 1 cm is associated with an abrupt change in \( v \) from positive to negative, suggesting that a yielding point of compaction was reached in this zone and lateral displacements are necessary to accommodate deformation, leading ultimately to voids lost between particles and the formation of a preferential fabric.

In summary, the results obtained with the combination of SP and FP using a single camera combined with top-view photographic analysis, allowed a high-resolution characterization of a granular-media surface during deformation. The flow of individual (or groups) constituents, local heterogeneity and discontinuity of strain can be analyzed at the grain scale or taking representative areas of the surface as a mechanically continuous material. The present experimental results may serve as a guide to further theoretical and numerical modeling.

5. Conclusions

Three-dimensional displacement of optically rough specimens subjected to deformation was measured by the use of a combination of fringe projection and speckle photography. The former yielded the out-of-plane displacement component whereas the latter the 2D in-plane displacement. The use of only one camera allowed us to avoid any calibration procedures which are usually needed with stereoscopic methods. The technique was applied to the measuring of the spatial and temporal evolution of 3D displacement fields for a deforming granular surface that represented a model of the Earth’s crust when subjected to compressive forces. It was found that the technique is well suited for this type of analysis and applications in other fields of engineering are envisaged.

The detail and resolution achieved permitted the experimental survey of the micromechanics of a surface of dry and non-cohesive granular media. It permitted us to identify stochastic particle mechanics and micro-inertia effects (vortex-like structures) in quasi-static flow conditions. Our experimental results may serve as a guide for further granular-media studies mainly in the theoretical and numerical simulations helping to constrain the underlying constitutive theory.

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