A folded wedge model for determination of true stratigraphic thickness in directional wells

Shunshan Xu* and A.F. Nieto-Samaniego

Universidad Nacional Autónoma de México, Centro de Geociencias, Apartado Postal 1-742, Querétaro, Qro., 76001, México
E-mail: sxu@dragon.geociencias.unam.mx
E-mail: afns@dragon.geociencias.unam.mx
*Corresponding author

J.M. Grajales-Nishimura, L.G. Velasquillo-Martínez and G. Murillo-Muñetón

Instituto Mexicano del Petróleo, No.152, Col. San Bartolo Atepehuácan, C.P. 07730, México, D.F., México
E-mail: mgrajal@imp.mx
E-mail: lgvelas@imp.mx
E-mail: gmurill@imp.mx

J. García-Hernandez

Petróleos Mexicanos Exploración y Producción Región Marina NE, Activo Cantarell (PEP-RMNE), Mexico
E-mail: jgarciah@pep.pemex.com

Abstract: For exploration and development the geologists and engineers try to know the true stratigraphic thickness \( t \). The monoclinal and folded bed models are published methods for \( t \) calculation. Both models assume parallel beds which can lead to significant deviations. In order to calculate the values of \( t \) considering the effects of both folding and stratigraphic variation we propose the ‘folded wedge model’. Three are considered: \( \beta_1 \), the bed dip at the point where the well enters; \( \beta_2 \), the bed dip at the point where the well leaves; and \( \alpha \), the deviated angle of borehole (angle between vertical and borehole). Three values of \( t \) can be obtained: the value measured normal to top \( (t_6) \); the value measured normal to bottom \( (t_7) \); the average value measured normal to bottom and top \( (t_8) \). The folded wedge model is applied to the Cantarell oil field complex obtaining more reasonable values of \( t \) than using the existing methods. [Received: August 16, 2009; Accepted: February 10, 2010]

Keywords: true stratigraphic thickness; wedging bed model; folded wedge model; absolute deviation.

Biographical notes: Xu Shunshan is a Structural Geologist with a broad experience in basin and petroleum geology, large-scale structures and fault dynamics. He worked on petroleum exploration and exploitation at the Mexican Institute of Petroleum from 2002 to 2007. He is currently working at the National University of Mexico. His research and technical work has been documented through in more than 30 reviewed publications.

Angel F. Nieto-Samaniego received his PhD in Geology in 1995. He has worked 25 years as a Researcher and a Professor of Structural Geology at the National University of Mexico. In 1997–1998, he was the President of the Mexican Geological Society. Currently, he is the Editor of three geological journals. Additionally, he is a member of the editorial advisory board of the Journal of Structural Geology.

José Manuel Grajales-Nishimura is now working in a multidisciplinary team on geological characterisation of carbonate oil reservoir at the Instituto Mexicano del Petróleo. He is responsible for diagenetic studies in order to characterise the origin and distribution of porosity in cretaceous carbonate rocks in Southern Mexico. He is currently working on sedimentology and stratigraphy of K-T boundary sedimentary sequences. He is a member of the GSA, and Mexican technical associations such as the AMGP, and SGM. His work has been published in about 24 technical papers.

L.G. Velasquillo-Martinez received his PhD from the Institut de Physique du Globe de Paris, France. For several years, his research interests include the neotectonic and seismic hazard of Isthmus of Tehauentepec in South of Mexico. He joined the Instituto Mexicano del Petróleo, Mexico, in 2000 as a Petroleum Geophysics and has worked several research projects in petroleum exploration and reservoir studies. His current areas of interest are in the characterisation of full field fracture distribution in carbonate rocks and the implications for the static reservoir characterisation.

Gustavo Murillo-Muñetón earned his MS degree in the University of Southern California and his PhD in Texas A&M University. He works for the Instituto Mexicano del Petroleo at Mexico City since 1984. His research areas include sedimentology, stratigraphy and diagenesis, mainly of carbonate systems. He carries out currently research and applied projects on petroleum exploration and exploitation from diverse Mexican basins. Additionally, he is a part-time teacher in the Instituto Politécnico Nacional where he teaches at the Sección de Graduados of the Escuela Superior de Ingeniería y Arquitectura-Unidad Ticomán.

Jesús Garcia Hernández is a Petroleum Geologist who has worked for PEMEX for the last 25 years. He received his degree in Petroleum Geology from the Instituto Tecnológico de Cd. Medero and MBA degree from Universidad Autónoma del Carmen-Tulane. He has extensive experience in prospect evaluation and production in the Cantarell Asset. Among many publications, one article published in the AAPG Bulletin in 2005 won the Wallace E. Pratt Award. He is an Asset Submanager in the Cantarell Asset.
1 Introduction

True stratigraphic thickness is the thickness of stratigraphic unit measured normal to the bedding surface (Bateman and Konen, 1979; Holt et al., 1977). True thickness can be derived from information determined by the dipmeter (e.g. Norman and Thibodaux, 1964). Some other methods such as borehole images, or borehole geophysical logs can also determine the stratigraphic thickness. The log thickness of a given stratigraphic interval can be thicker, equal to, or thinner than that seen in a vertical well drilled through the same stratigraphic section because of formation dip (e.g. Travis, 1979; Bateman and Konen, 1979). As long as dips and deviations do not exceed a few degrees, the simple vertical-horizontal case is approximated closely enough by the actual logs. However, if borehole deviations and bed dips exceed about ten degrees, corrections are needed. Traditional method of correction is based on the assumption that the bed thickness does not change from the point that the well enters to the point that the well leaves (e.g. Bateman and Konen, 1979; Groshong, 1999; Tearpock and Bischke, 2003). However, this assumption is approximate, because the vertical and horizontal stratigraphic changes are generally of subtle nature (e.g. Drummond and Wilkinson, 1996; Drummond, 1999; Dadlez, 2003). Xu et al. (2007) proposed a folded bed model to calculate the true stratigraphic thickness. This model considers the cases where the attitudes of the bed are different between the point where the well enters and the point where the well leaves due to folding. This model is still a parallel bed model. If the upper surface and lower surface for the sedimentary beds are not parallel to each other, the folded parallel bed model is not appropriate to calculate the true stratigraphic thickness.

For offshore exploration and development, multiple deviated wells are commonly needed in the fixed platforms. Therefore, the true stratigraphic thickness is important in inclined beds and deviated wells, since reservoir volume and isopach depend on the true stratigraphic thickness and not on the directly measured thickness. In this paper, a general method is proposed to calculate the true stratigraphic thickness considering the effects of both stratigraphic variation and folding.

2 Wedging bed model for calculation of the value of $t$

Before starting the wedging bed model, we review on the published parallel bed models (e.g. Lindqvist, 1982; Lisle, 2003; Evenick, 2008). At first, the monoclinal bed model is reviewed here. This model does not consider the changes of neither stratigraphic thickness nor the attitude in the vicinity of the region where the well crosses.

Under the assumption that both the bed azimuth and the well azimuth are the same, the values of $t$ can be calculated as the following equation (Xu et al., 2007)

$$t = h_m \cos (\beta + \alpha)$$

(1)

where $h_m$ is the distance between the point where the well enters and the point where the well leaves, $\beta$ is the bed dip, and $\alpha$ is the deviation angle of borehole.

Similarly, if the bed azimuth is opposite to the well azimuth, the values of $t$ can be calculated as following equation

$$t = h_m \cos (\beta - \alpha)$$

(2)
On the other hand, for the concentric folded bed model, it is considered the change of bed attitude only due to folding or flexturing but not due to stratigraphic variation in the region where the well crosses (Xu et al., 2007). If the bed azimuth is the same as the well azimuth, the true stratigraphic thickness is

\[ t_2 = h_m \frac{\cos(\beta_2 + \beta_1 + \alpha)}{2 \cos(\beta_2 - \beta_1)} \]  

(3)

where \( \beta_1 \) is the bed dip at the point where the well enters and \( \beta_2 \), the bed dip at the point where the well leaves.

Likewise, if the bed azimuth is opposite to the well azimuth, the value of \( t \) can be estimated by

\[ t_2 = h_m \frac{\cos(\beta_2 + \beta_1 - \alpha)}{2 \cos(\beta_2 - \beta_1)} \]  

(4)

For establishing a ‘folded wedge model’, we first introduce the ‘wedging bed model’. The wedging bed model considers only stratigraphic variation in the region the well crosses but not the change of bed attitude due to folding or flexturing. Three cases are considered according to the surface referenced. First, the true thickness is measured normal to the top bed surface. Second, the measurement is made normal to the bottom surface. Third, the average value measured normal to bottom and top.

On the other hand, the value of \( \alpha \) needs to be corrected if the dip direction on the top of the bed is not the same as the well azimuth. The apparent deviation angle \( (\alpha') \) in the dip direction of the bed can be calculated by: \( \alpha' = \arctan(\tan(\alpha \cos \gamma)) \), where \( \gamma \) is the intersection angle between the strike of the top surface of the bed and the well azimuth. Note that the value of \( \alpha' \) is negative when \( \gamma = 90–270^\circ \). Thus this equation can be applied to the following models in the case of deviated wells.

### 2.1 Measurement normal to the top surface for wedging bed model

Figure 1 shows that both the top and bottom of the bed are inclined. The tilting of the bed would be due to sedimentation, deformation etc. (e.g., Pallesen and Ottesen, 2002). For calculation, we consider two combinations between the well azimuth and bed azimuth. Figure 1(a) shows that the well azimuth is the same as the bed azimuth. The true stratigraphic thickness at point A can be estimated by the Law of Sines in triangle ABC.

\[ t_3 = AC = \frac{AB \sin(90 - \beta_2 - \alpha))}{\sin(90 - \beta_1 + \beta_2)} = h_m \frac{\cos(\beta_2 + \alpha)}{\cos(\beta_2 - \beta_1)} \]  

(5)

Likewise, if the well azimuth is opposite to the bed azimuth [Figure 1(b)], for the triangle ABC, we can obtain
A folded wedge model for determination of true stratigraphic thickness

\[ t_3 = AC = \frac{AB \sin(90 + \beta_2 - \alpha)}{\sin(90 - \beta_2 + \beta_1)} = \frac{h_m \cos(\beta_2 - \alpha)}{\cos(\beta_2 - \beta_1)}. \]  

(6)

Specially, if the well is vertical, equations (5) and (6) can be written as

\[ t_3 = \frac{h_m \cos \beta_2}{\cos(\beta_2 - \beta_1)}. \]  

(7)

Figure 1  Sketches showing the wedging bed models

Notes: In (a) and (b), the bed thickness is measured normal to the top boundary. For (a), the well azimuth is the same as the azimuth of the bed. For (b), the well azimuth is opposite to the azimuth of the bed. In (c) and (d), the bed thickness is measured normal to the bottom. For (c), the well azimuth is the same as the azimuth of the bed. For (d), the well azimuth is opposite to the azimuth of the bed.

2.2 Measurement normal to the bottom

In this case, the thickness is measured normal to the lower bed boundary, where the well leaves that bed. If the borehole is drilled by deviated angle, also, there are two combinations between the well azimuth and bed azimuth. Figure 1(c) shows that the well azimuth is the same as the azimuth of the bed. The true bed thickness \( t_4 \) can be estimated according to the Law of Sines. For the right triangle ACB, \( \angle ACB = 90^\circ \), \( \angle ABC = 90 - \alpha - \beta_2 \), therefore

\[ t_4 = AC = AB \sin(90 - \alpha - \beta_2) = h_m \cos(\alpha + \beta_2). \]  

(8)
Similarly, when the well azimuth is opposite to the azimuth of the bed [Figure 1(d)], the true bed thickness \( t_4 \) can be estimated as

\[
t_4 = AC = AB \sin(90 - \alpha + \beta_2) = h_m \cos(\beta_2 - \alpha). \tag{9}
\]

Particularly, if the well is vertical, equations (8) and (9) can be written as

\[
t_4 = h_m \cos \beta_2. \tag{10}
\]

### 2.3 Average value measured normal to bottom and top

According to equations (8) and (5), or equations (9) and (6), we can obtain that

\[
t_4 / t_3 = \cos(\beta_2 - \beta_1) < 1. \tag{11}
\]

This equation indicates that the true thickness measured normal to bottom is always smaller than that measured normal to top. In some case, the more realistic measure may be somewhere between the two special cases. Therefore, approximately, their average value can be used as the true stratigraphic thickness, which may be more meaningful than the values of the two special cases. In this way, the true stratigraphic thickness can be calculated by \( t_5 = (t_4 + t_3)/2 \). If the well azimuth is the same as the azimuth of the bed, the value of \( t_5 \) is

\[
t_5 = \frac{h_m \cos(\beta_2 + \alpha) \left[1 + \cos(\beta_2 - \beta_1)\right]}{2 \cos(\beta_2 - \beta_1)}. \tag{12}
\]

If the well azimuth is opposite to the azimuth of the bed, the value of \( t_5 \) will be

\[
t_5 = \frac{h_m \cos(\beta_2 - \alpha) \left[1 + \cos(\beta_2 - \beta_1)\right]}{2 \cos(\beta_2 - \beta_1)}. \tag{13}
\]

Specially, for the vertical boreholes, the true stratigraphic thickness is calculated by

\[
t_5 = \frac{h_m \cos(\beta_2) \left[1 + \cos(\beta_2 - \beta_1)\right]}{2 \cos(\beta_2 - \beta_1)}. \tag{14}
\]

### 3 Folded wedge model

Generally, when \( \beta_2 \neq \beta_1 \) (\( \beta_2 > \beta_1 \) or \( \beta_2 < \beta_1 \)), the bed can be considered as a folded and stratigraphic variation bed. That is to say, both the stratigraphic variation in the region where the well crosses and the change of bed attitude due to folding or flexuring should be considered. We define this model as folded wedge model. In this case, the true stratigraphic thickness can not be estimated by accurate mathematical formula (Figure 2), but its approximate value can be obtained. In Figure 2, AF is the true stratigraphic thickness \( t_3 \) measured normal to the top surface for the wedging bed model. AD is the true stratigraphic thickness \( t_2 \) for the folded bed model. The true stratigraphic thickness (AG) for the folded wedge model is always between the values of \( t_2 \) and \( t_3 \) [Figures 2(a), 2(b), 2(c) and 2(d)]. Therefore, the true value can be approximately estimated by

\[
t_6 = (t_5 + t_2)/2. \tag{15}
\]
When the well azimuth is the same as the azimuth of the bed, according to equations (3) and (5), we have

\[
t_6 = \frac{h_m}{2} \left[ \frac{\cos(\beta_2 + \beta_1) + \alpha}{\cos \frac{\beta_2 - \beta_1}{2}} + \frac{\cos(\beta_2 + \alpha)}{\cos(\beta_2 - \beta_1)} \right].
\]

(16)

**Figure 2** Sketches showing folded wedge model in which the true thickness is measured normal to the top surface

Notes: AF \( (t_3) \) is the true stratigraphic thickness for the wedging bed model, AD \( (t_2) \) for the folded bed model. AC is the true stratigraphic thickness for the monoclinal bed model. The approximate estimation of the true stratigraphic thickness \( (AG) \) for the folded wedge model is \( t_6 = (t_3 + t_2)/2 \). (a) in the case of synform, the well with the same azimuth as the bed, and \( \beta_2 < \beta_1 \). (b) in the case of synform, the well with the opposite azimuth to the bed, and \( \beta_2 > \beta_1 \). (c) in the case of antiform, the well with the same azimuth as the bed, and \( \beta_2 > \beta_1 \). (d) in the case of antiform, the well with the opposite azimuth to the bed, and \( \beta_2 < \beta_1 \).

Likewise, when the well azimuth is opposite to the azimuth of the bed, according to equations (4) and (6), the value of \( t_6 \) is
\[ t_6 = \frac{h_m}{2} \left\{ \frac{\cos(\frac{2\beta_2 + \beta_1}{2}) - \alpha \cos(\frac{2\beta_2 - \alpha}{2})}{\cos(\frac{2\beta_2 - \alpha}{2}) + \cos(\frac{2\beta_2 - \beta_1}{2})} \right\}. \] 

(17)

Particularly, for the vertical wells \((\alpha = 0)\), the value of \(t_6\) can be calculated as

\[ t_6 = \frac{h_m}{2} \left\{ \frac{\cos(\frac{2\beta_2 + \beta_1}{2})}{\cos(\frac{2\beta_2 - \alpha}{2}) + \cos(\frac{2\beta_2 - \beta_1}{2})} \right\}. \] 

(18)

On the other hand, if the true stratigraphic thickness is measured normal to the bottom surface [Figures 3(a), 3(b), 3(c) and 3(d)], the approximate value of \(t\) for the folded wedge model can be \(t_7 = (t_4 + AD')/2\). Note that here \(AD'\) is not equal to \(AD\) (the value of \(t_2\)) for the folded bed model. According to Appendix A, in the case of a synform, if the well azimuth is the same as the azimuth of the bed [Figure 3(a)], the value of \(AD'\) is

\[ AD' = \frac{h_m \cos(\beta_2 + \alpha + \phi)}{\cos(\phi)}. \] 

(19)

where the value of \(\phi\) is

\[ \frac{1}{2} \arcsin \left( \frac{\sin(\alpha + \beta_2) \sin(\beta_1 - \beta_2)}{\sin(\alpha + \beta_1)} \right). \]

Then, from equations (8) and (19), the value of \(t_7\) can be

\[ t_7 = \frac{h_m}{2} \left\{ \frac{\cos(\beta_2 + \alpha + \phi) \cos(\phi)}{\cos(\phi)} + \cos(\alpha + \beta_2) \right\}. \] 

(20)

On the other hand, when the well azimuth is opposite to the azimuth of the bed [Figure 3(b)], according to Appendix A, the value of \(AD'\) is

\[ AD' = \frac{h_m \cos(\alpha + \phi - \beta_2)}{\cos(\phi)}. \] 

(21)

where the value of \(\phi\) is

\[ \frac{1}{2} \arcsin \left( \frac{\sin(\alpha - \beta_2) \sin(\beta_2 - \beta_1)}{\sin(\alpha - \beta_1)} \right). \]

Then, by combining equations (9) and (21), the value of \(t_7\) is

\[ t_7 = \frac{h_m}{2} \left\{ \frac{\cos(\alpha + \phi - \beta_2) \cos(\phi)}{\cos(\phi)} + \cos(\alpha - \beta_2) \right\}. \] 

(22)

In the case of an antiform, if the well azimuth is the same as the azimuth of the bed [Figure 3(c), Appendix A], the value of \(AD'\) is

\[ AD' = \frac{h_m \cos(\beta_2 + \alpha - \phi)}{\cos(\phi)}, \] 

(23)

where \(\phi = (\beta_2 - \beta_1)/2\).

Therefore, the value of \(t_7\) can be inferred from equations (8) and (23)
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\[ t_7 = \frac{h_m}{2} \left\{ \frac{\cos(\alpha - \varphi + \beta_2)}{\cos \varphi} + \cos(\alpha + \beta_2) \right\} \]

(24)

Likewise, when the well azimuth is opposite to the azimuth of the bed [Figure 3(d)], the value of \( t_7 \) is determined by equations

\[ t_7 = \frac{h_m}{2} \left\{ \frac{\cos(\alpha - \varphi - \beta_2)}{\cos \varphi} + \cos(\alpha - \beta_2) \right\}, \]

(25)

where \( \varphi = (\beta_1 - \beta_2)/2 \).

Figure 3 Sketches showing folded wedge model in which the true thickness is measured normal to the bottom of bed

Notes: \( AF' (t_4) \) is the true stratigraphic thickness for the wedging bed model, and \( AD \) is that for the folded bed model. Note that \( AD \neq AD' \). The estimation of the true stratigraphic thickness for the folded wedge model is \( AG' \). Because \( AD \neq AD' \), the value of \( AD' \) should be calculated additionally (see Appendix A). (a) in the case of synform, the well with the same azimuth as the bed, and \( \beta_2 < \beta_1 \) (b) in the case of synform, the well with the opposite azimuth to the bed, and \( \beta_2 > \beta_1 \) (c) in the case of antiform, the well with the same azimuth as the bed, and \( \beta_2 > \beta_1 \) (d) in the case of antiform, the well with the opposite azimuth to the bed, and \( \beta_2 < \beta_1 \).
If the average value from the measurements normal to upper and lower surfaces is used as the true thickness, the approximate value of $t$ for the folded wedge model may be $t_8 = (t_7 + t_6)/2$. In this way, for a synform, in which the well azimuth is the same as the azimuth of the bed, by combining equations (16) and (20):

$$t_8 = \frac{h_m}{2} \left\{ \frac{\cos\left(\frac{\beta_2 + \beta_1}{2} + \alpha\right)}{\cos\left(\frac{\beta_2 - \beta_1}{2}\right)} + \frac{\cos(\beta_2 + \alpha + \varphi)}{\cos(\beta_2 - \beta_1)} + \frac{\cos(\alpha + \beta_2)}{\cos \varphi} \right\}, \quad 26)$$

and when the well azimuth is opposite to the azimuth of the bed, by combining equations (17) and (22) the value of $t_8$ is

$$t_8 = \frac{h_m}{2} \left\{ \frac{\cos\left(\frac{\beta_2 + \beta_1}{2} - \alpha\right)}{\cos\left(\frac{\beta_2 - \beta_1}{2}\right)} + \frac{\cos(\beta_2 - \alpha)}{\cos(\beta_2 - \beta_1)} + \frac{\cos(\alpha + \beta_2)}{\cos \varphi} \right\}. \quad 27)$$

On the other hand, in the case of an antiform, when the well azimuth is the same as the azimuth of the bed, by combining equations (16) and (24) we get:

$$t_8 = \frac{h_m}{2} \left\{ \frac{\cos\left(\frac{\beta_2 + \beta_1}{2} + \alpha\right)}{\cos\left(\frac{\beta_2 - \beta_1}{2}\right)} + \frac{\cos(\beta_2 + \alpha - \varphi)}{\cos(\beta_2 - \beta_1)} + \frac{\cos(\alpha + \beta_2)}{\cos \varphi} \right\}. \quad 28)$$

In this case, if the well azimuth is opposite to the azimuth of the bed, by combining equations (17) and (25) the value of $t_8$ is

$$t_8 = \frac{h_m}{2} \left\{ \frac{\cos\left(\frac{\beta_2 + \beta_1}{2} - \alpha\right)}{\cos\left(\frac{\beta_2 - \beta_1}{2}\right)} + \frac{\cos(\alpha - \beta_2)}{\cos(\beta_2 - \beta_1)} + \frac{\cos(\alpha - \beta_2)}{\cos \varphi} \right\}. \quad 29)$$

### 4 Influence factors of the value of $t$ for the folded wedge model

From equations (16) to (29), we can see that the value of $t$ is dependent on the variables $\beta_2$, $\beta_1$, $\alpha$, and $\varphi$, but the value of $\varphi$ is not an independent variable (see Appendix A). Figure 4 shows the relationship between the values of $\beta_2$, $\beta_1$, and $\alpha$ respectively, and the value of $t_8$. For the constant values of $\beta_2$ and $\alpha$, when $\beta_1$ is smaller, it has negative relationship with the value of $t_8$, whereas, when $\beta_1$ is larger, it has positive relationship [Figures 4(a) and 4(b)]. The inflection point between the negative and positive relationship is dependent on the values of $\alpha$. For the constant values of $\beta_1$ and $\alpha$, in the case where the bed azimuth is opposite to the well azimuth, there are two relationships between $\beta_2$ and the value of $t_8$ [Figure 4(c)]. When $\alpha$ is larger than $\beta_1$ (for the example, $\beta_1 = 25^\circ$), the value of $\beta_2$ has positive relationship with the value of $t_8$, whereas, when $\alpha$ is smaller than $\beta_1$, it has negative relationship [Figure 4(c)]. In the case where the bed azimuth is same as the well azimuth, the value of $\beta_2$ has negative relationship with the
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Given constant values of $\beta_2$ and $\beta_1$, in the case where the bed azimuth is opposite to the well azimuth, for smaller values of $\alpha$, it has positive relationship with the value of $t_6$, whereas, for larger values of $\alpha$, it has negative relationship [Figure 4(e)]. Additionally, in the case where the bed azimuth is same as the well azimuth, the value of $\alpha$ decreases with the increase of the value of $t_6$ [Figure 4(f)].

Figure 4  Change tendency of the value of $t_6$

<table>
<thead>
<tr>
<th>The well azimuth is opposite to the bed azimuth</th>
<th>The well azimuth is same as the bed azimuth</th>
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<tbody>
<tr>
<td><img src="image1" alt="Figure 4(a)" /></td>
<td><img src="image2" alt="Figure 4(b)" /></td>
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<td><img src="image4" alt="Figure 4(d)" /></td>
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<td><img src="image5" alt="Figure 4(e)" /></td>
<td><img src="image6" alt="Figure 4(f)" /></td>
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</table>

Notes: In (a) and (b), the relationship between $\beta_1$ and $t_6$ is shown, when $h_m = 200$ m, $\beta_2 = 25^\circ$ are given. In (c) and (d), the relationship between $\beta_2$ and $t_6$ is shown, when $h_m = 200$ m, $\beta_1 = 25^\circ$ are known. In (e) and (f), the relationship between $\alpha$ and $t_6$ is shown, when $h_m = 200$ m, $\beta_2 = 20$, and $\beta_1 = 30^\circ$ are given.

The relationship between the values of $\beta_2$, $\beta_1$, and $\alpha$ respectively, and the value of $t_7$ is shown in Figure 5. For the given values of $\beta_2$ and $\alpha$, the relationship between $t_7$ and $\beta_1$ is complicated [Figures 5(a) and 5(b)]. The forms of curve are dependent on the values of $\alpha$. For the constant values of $\beta_1$ and $\alpha$, in the case where the bed azimuth is opposite to the well azimuth [Figure 5(c)], when $\beta_2$ is smaller, it has positive relationship with the value of $t_7$, whereas, when $\beta_2$ is larger, it has positive relationship. The inflection point between the negative and positive relationship is dependent on the values of $\alpha$ [Figure 5(c)]. On the other hand, when the bed azimuth is same as the well azimuth [Figure 5(d)], the value...
of \(t_7\) has negative relationship with the value of \(\beta_2\). For the constant values of \(\beta_2\) and \(\beta_1\), the variation of \(t_7\) in Figures 5(e) and 5(f) is similar to the \(t_6\) in Figures 4(e) and 4(f).

**Figure 5** Variation of the value of \(t_7\)

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<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Notes: In (a) and (b), variation of \(t_7\) with \(\beta_1\) and is shown for the given values of \(h_m = 200\) m and \(\beta_2 = 25^\circ\). In (c) and (d), variation of \(t_7\) with \(\beta_2\) is shown for the given values of \(h_m = 200\) m and \(\beta_1 = 25^\circ\). In (e) and (f), the change tendency of \(t_7\) with \(\alpha\) is shown, when \(h_m = 200\) m, \(\beta_2 = 20\), and \(\beta_1 = 30^\circ\) are given.

The variation of \(t_8\) is sketched in Figure 6. For the constant values of \(\beta_2\) and \(\alpha\), in the case where the bed dip direction is opposite to the well azimuth [Figure 6(a)], the form of curves for \(t_8\) is dependent on the value of \(\alpha\). Generally, the value of \(\beta_2\) has positive relationship with the value of \(t_8\), but for the larger value of \(\alpha\), there are local waves, for example, the curve for \(\alpha = 40\) in Figure 6(a). In the case where the bed azimuth is same as the well azimuth, when \(\beta_1\) is smaller, it has negative relationship with the value of \(t_8\), whereas when \(\beta_1\) is larger, it has positive relationship [Figure 6(b)]. The inflection point between the negative and positive relationship changes due to values of \(\alpha\). For the constant values of \(\beta_1\), the variation of the value of \(t_8\) is similar to the value of \(t_7\) [Figures 6(c) and 6(d)]. Given constant values of \(\beta_2\) and \(\beta_1\), the change of the value of \(t_8\) in Figures 6(e) and 6(f) is also similar to the value of \(t_7\) in Figures 5(e) and 5(f).
Figure 6  Change characteristics of the value of $t_6$ with the values of $\beta_1$, $\beta_2$, and $\alpha$.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>The well azimuth is opposite to the bed azimuth</td>
<td><img src="image1.png" alt="Graphs" /></td>
</tr>
<tr>
<td>The well azimuth is the same as the bed azimuth</td>
<td><img src="image2.png" alt="Graphs" /></td>
</tr>
</tbody>
</table>

Notes: In (a) and (b), change of $t_6$ with $\beta_1$ is shown for the given values of $h_m = 200$ m and $\beta_2 = 25^\circ$. In (c) and (d), change of $t_6$ with $\beta_2$ is shown for the given values of $h_m = 200$ m and $\beta_1 = 25^\circ$. In (e) and (f), the change tendency of $t_6$ with $\alpha$ is shown, when $h_m = 200$ m, $\beta_2 = 20$, and $\beta_1 = 30^\circ$ are given.

By comparing three values of thickness calculated from the three special cases (Figures 4, 5 and 6), three characteristics can be seen:

1. The value of $t_6$ is largest, and the value of $t_7$ is smallest.
2. The values of $\alpha$, $\beta_2$, $\beta_1$, respectively, have different relationships with the value of $t_6$ or $t_7$ or $t_8$.
3. The curves of three parameters ($t_6$, $t_7$ and $t_8$) in the case where the bed azimuth is opposite to the well azimuth are more complicated than in the case where the bed azimuth is opposite to the well azimuth.

5  Deviation of the value of $t$ for the existing models comparing with the folded wedge model

In order to understand the difference of the values of $t$ between the folded wedge model and the two existing models (monoclonal bed model and folded bed model), the absolute
deviation of the value of \( t \) for the monoclinal bed model is defined as \( A_{ti} \), and \( A_{tf} \) is for the folded bed model. They are written as following equations

\[
A_{ti} = t_8 - t_1, \quad (30) \\
A_{tf} = t_8 - t_2. \quad (31)
\]

**Figure 7** Curves of the value of \( A_{ti} \)

Notes: In (a) and (b), change of \( A_{ti} \) with \( \beta_1 \) is shown for the given values of \( h_m = 200 \) m and \( \beta_2 = 25^\circ \). In (c) and (d), change of \( A_{ti} \) with \( \beta_2 \) is shown for the given values of \( h_m = 200 \) m and \( \beta_1 = 25^\circ \). In (e) and (f), the change tendency of \( A_{ti} \) with \( \alpha \) is shown, when \( h_m = 200\)m, \( \beta_2 = 20 \), and \( \beta_1 = 30^\circ \) are given.

Figures 7(a) and 7(b) show that there is a non-linear relationship between the value of \( A_{ti} \) and \( \beta_1 \), when the values of \( \alpha \) and \( \beta_2 \) are given. In the case where the bed azimuth is opposite to the well azimuth [Figure 7(a)], the tendency of curves is dependent on the value of \( \alpha \). For \( \alpha = 10^\circ \), the value of \( A_{ti} \) increases with the increase of \( \beta_1 \). For \( \alpha = 20^\circ \), \( 30^\circ \), and \( 40^\circ \), when \( \beta_1 \) is smaller, the value of \( A_{ti} \) has negative relationship with \( \beta_1 \), whereas, when \( \beta_1 \) is larger, a positive relationship is presented. If the bed azimuth is same
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as the well azimuth [Figure 7(b)], a positive relationship between $\beta_1$ and $A_{ti}$ is shown. On the other hand, if the values of $\alpha$ and $\beta_1$ are given, the variation of $A_{ti}$ is also complicated. If the bed azimuth is opposite to the well azimuth, two types of curve can be seen [Figure 7(c)]. For $\alpha = 10^\circ$, the value of $A_{ti}$ decreases with increasing the value of $\beta_2$. For $\alpha = 20^\circ$, $30^\circ$, and $40^\circ$, the left segment of the curves shows progressive increase and the right segment shows progressively decrease. When the bed azimuth is same as the well azimuth, the value of $A_{ti}$ decreases with increasing the value of $\beta_2$ [Figure 7(d)]. For the constant values of $\beta_1$ and $\beta_2$, if the bed azimuth is opposite to the well azimuth, the parameter $\alpha$ is positively related to the value of $A_{ti}$ [Figure 7(e)]. However, the parameter $\alpha$ shows a negative relationship with the value of $A_{ti}$ [Figure 7(f)].

Figure 8  Curves of the value of $A_{ti}$

The well azimuth is opposite to the bed azimuth | The well azimuth is the same the bed azimuth

Notes: In (a) and (b), change of $A_{ti}$ with $\beta_1$ is shown for the given values of $h_m = 200$ m and $\beta_2 = 25^\circ$. In (c) and (d), change of $A_{ti}$ with $\beta_2$ is shown for the given values of $h_m = 200$ m and $\beta_1 = 25^\circ$. In (e) and (f), the change tendency of $A_{ti}$ with $\alpha$ is shown, when $h_m = 200$ m, $\beta_2 = 20$, and $\beta_1 = 30^\circ$ are given.
Difference of the value of $t$ between the folded wedge model and the folded bed models ($A_{tf}$) in Figure 8 shows similar change characteristics to the value of $A_{ti}$ in Figure 7. There is a little difference only in the form of curves between two figures. By comparing two parameters ($A_{tf}$ and $A_{ti}$), five characteristics are concluded:

1. the values of $A_{tf}$ and $A_{ti}$ can be either larger than or smaller than zero
2. the relationships between $A_{tf}$ or $A_{ti}$ and $\alpha$, $\beta_2$, $\beta_1$, and $h_{in}$, respectively, are different
3. for the same values of $\alpha$, $\beta_2$, $\beta_1$, and $h_{in}$, the value of $A_{tf}$ is different from the value of $A_{ti}$
4. the change range of $A_{tf}$ is smaller than that of $A_{ti}$, which indicates that the value of $t$ for the folded bed model, compared with the monoclinal bed model, is closer to that for the folded wedge model
5. the curves of two parameters ($A_{tf}$ and $A_{ti}$) in the case where the bed azimuth is opposite to the well azimuth are more complicated than in the case where the bed azimuth is opposite to the well azimuth.

6. Example

The folded model is applied to the Cantarell oil field complex in the southern Gulf of Mexico (offshore, Campeche). The interpretation of geologic and geophysical data suggested that Cantarell is a fold-thrust belt and duplex structure (e.g. Angeles et al. 1994; PEMEX, 1999; Xu et al. 2004). The stratigraphic units in this complex include Oxfordian, Tithonian, Cretaceous, and Tertiary system (Grajales et al., 2000). The dolomitised breccias in the Upper Cretaceous and Lower Palaeocene (BKT) are selected to calculate the true stratigraphic thickness. The attitudes of the top and bottom of this unite were obtained from the structure contour maps. The stratigraphic depth data of wells were obtained from PEMEX.

The boreholes are divided into two groups according to the deviated angles. Group 1 includes 161 boreholes with the deviated angel ($\alpha \neq 0$). Group 2 includes 23 vertical boreholes ($\alpha = 0$). The boreholes of group 1 are divided into two sub-groups according to the values of bed dips. The first subgroup has 25 boreholes with $|\beta_2 - \beta_1| > 10^\circ$. The second subgroup has 136 boreholes with $|\beta_2 - \beta_1| < 10^\circ$. For all boreholes, the average dips of BKT at the top is 17.3$^\circ$, at bottom is 18$^\circ$. The average deviated angle of the deviated wells is 32.3$^\circ$. For comparison, we calculated the values of $t_1$ for the monoclinal bed model, $t_2$ for the folded bed model, and $t_8$ for the folded wedge model, respectively. The average value of the true bed thickness for the folded wedge model is less than that for the monoclinal bed model and the concentric folded bed model in both vertical wells and deviated wells (Table 1). The results show that in general, the monoclinal bed model and folded bed model overestimate the true bed thickness.

By investigating the characteristics of values of $t$ for each model, we introduce the parameter of standard deviation, which can be written as
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\[ S = \sqrt{\frac{\sum t^2 - (\sum t)^2}{n - 1}}, \] (32)

where \( t \) is the values of the true stratigraphic thickness, \( n \) is total number of a dataset, \( S \) is standard deviation. This parameter describes the degree of variation of a dataset. Larger value indicates a larger degree of variation in a dataset. The results shown in Table 2 indicate that in all cases the values of \( S \) for \( t_8 \) are smaller than those for \( t_1 \) or \( t_2 \). These results imply that the change of the bed thickness obtained from the folded wedge model is more reasonable than the change from the monoclinal bed model or folded bed model.

**Table 1**  
Average value of \( t \) calculated by different models

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Number of data</th>
<th>( t_1 ) (m)</th>
<th>( t_2 ) (m)</th>
<th>( t_8 ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>161</td>
<td>257.2</td>
<td>255.3</td>
<td>254.6</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>213.3</td>
<td>208.2</td>
<td>206.3</td>
</tr>
<tr>
<td>C</td>
<td>136</td>
<td>265.3</td>
<td>263.2</td>
<td>263.0</td>
</tr>
<tr>
<td>D</td>
<td>23</td>
<td>280.2</td>
<td>278.9</td>
<td>278.3</td>
</tr>
</tbody>
</table>

Notes: Case of A – all deviated wells; case B – deviated wells with \( |\beta_2 - \beta_1| > 10^\circ \); case C – deviated wells with \( |\beta_2 - \beta_1| < 10^\circ \); case D – vertical wells.

**Table 2**  
Standard deviations of thickness datasets obtained from different models

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Number of data</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>161</td>
<td>67.9</td>
<td>67.4</td>
<td>67.1</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>99.5</td>
<td>72.8</td>
<td>71.8</td>
</tr>
<tr>
<td>C</td>
<td>136</td>
<td>64.5</td>
<td>63.1</td>
<td>63.0</td>
</tr>
<tr>
<td>D</td>
<td>23</td>
<td>81.5</td>
<td>78.7</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Notes: \( S_1 \) - standard deviations for \( t_1 \); \( S_2 \) – standard deviations for \( t_2 \); \( S_8 \) – standard deviations for \( t_8 \); case A – all deviated wells; case B – deviated wells with \( |\beta_2 - \beta_1| > 10^\circ \); case C – deviated wells with \( |\beta_2 - \beta_1| < 10^\circ \); case D – vertical wells.

**Figure 9**  
Histogram of the values of \( |t_8 - t_1| \) and \( |t_8 - t_2| \) for the formation BKT in the Cantarell
On the other hand, from Figure 9, we can compare the values of \(|t_8 - t_1|\) and \(|t_8 - t_2|\). First, the range of \(|t_8 - t_1|\) is wider than that of \(|t_8 - t_2|\). Second, the number that ranges from 0 to 5 m for \(|t_8 - t_2|\) is more than that for \(|t_8 - t_1|\). These results indicate that the values of \(t\) for the folded bed model, comparing with the monoclinal bed model are closer to those for the folded wedge model.

**Figure 10** Isopach of the true stratigraphic thickness of the formation BKT in the Cantarell for the monoclinal bed model (see online version for colours)

Notes: The contour lines are in metres.

By using the data of \(t\), we can make isopach maps. As a result, three isopach maps of the true stratigraphic thickness for \(t_1\), \(t_2\), and \(t_8\) can be obtained (Figures 10, 11 and 12). There are local differences among three maps. For example, near point A, the area less than 200
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m for the monoclinal bed model is larger than 1.5 km², but that for the folded wedge model is less than 1.3 km². For further comparison, we can measure systematic data from the maps. In this way, we measure data from five sections in each map. Also, the standard deviations are calculated for each datasets from each section. The results are shown in Table 3. The average value (51.9) of standard deviation for the monoclinal bed model is larger than that for the folded bed model or folded wedge model. Once again, this indicates that the folded wedge model is better for calculation of the true stratigraphic thickness.

Figure 11  Isopach of the true stratigraphic thickness of the formation BKT in the Cantarell for the folded bed model (see online version for colours)

Notes: The contour lines are in metres.
Figure 12  Isopach of the true stratigraphic thickness of the formation BKT in the Cantarell for the folded wedge model (see online version for colours)

Notes: The contour lines are in metres.

Table 3  Standard deviations of thickness datasets measured from the sections of contour maps in Figs 10, 11, 12

<table>
<thead>
<tr>
<th>Type of model</th>
<th>Section PP_1</th>
<th>Section PP_2</th>
<th>Section PP_3</th>
<th>Section PP_4</th>
<th>Section PP_5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monoclinal</td>
<td>36.0</td>
<td>43.7</td>
<td>62.7</td>
<td>47.9</td>
<td>69.2</td>
<td>51.9</td>
</tr>
<tr>
<td>Folded</td>
<td>35.9</td>
<td>45.2</td>
<td>56.4</td>
<td>38.6</td>
<td>60.1</td>
<td>47.3</td>
</tr>
<tr>
<td>Synthetic</td>
<td>34.0</td>
<td>43.6</td>
<td>53.6</td>
<td>43.3</td>
<td>62.0</td>
<td>47.3</td>
</tr>
</tbody>
</table>
7 Conclusions

When the attitude of the bed is not the same at the point the well enters as it is when the well leaves the bed, the folded wedge model should be considered to calculate the values of \( t \). For calculation, the values in three cases are considered:

1. the value normal to bottom
2. the value normal to top
3. average of values normal to the bottom and top.

In each case, two combinations between the bed azimuth and the well azimuth are considered separately. The established equations reveal that the value of \( t_b \) for the case 1 is largest, and that the value of \( t_i \) for case 2 is smallest. The main influence factors of the values of \( t \) are the values of \( \alpha \), \( \beta_2 \), \( \beta_i \), which, respectively, have different relationships with the value of \( t_b \) or \( t_i \) or \( t_s \). The changes of three values \( (t_b, t_i, t_s) \) in the case where the bed azimuth is opposite to the well azimuth are more complicated than in the case where the bed azimuth is opposite to the well azimuth.

By comparing the folded wedge model with the existing models, we obtain the following results. First, the values of \( t \) for the folded wedge model can be either larger than or smaller than those for the monoclinal bed model or folded bed model. Second, the deviation of \( t \) for the monoclinal bed model is larger than that for the folded bed model. This suggests that the value of \( t \) for the folded bed model, compared with the monoclinal bed model, is closer to that for the folded wedge model. Third, the difference of \( t \) between the folded wedge model and two existing model in the case where the bed azimuth is opposite to the well azimuth are more complicated than in the case where the bed azimuth is opposite to the well azimuth.

For application, a case study from the Cantarell oil field complex in the southern Gulf of Mexico (offshore, Campeche) is given to test this method. The standard deviations of datasets for \( t \) from the folded wedge model are smaller than those from the monoclinal bed mode and folded bed model. This indicates that the folded wedge model can obtain more reasonable data. In general, the monoclinal bed mode and folded bed model will overestimate the true stratigraphic thickness.

Acknowledgements

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References


**Appendix A**

For calculation of the value of AD’, the cases of antiform and synform should be considered separately. Figure 3(a) shows the case where the well azimuth is the same as the azimuth of the bed in a synform. The value of ∠D’BF’ is assumed to be φ. This angle is a tangent chord angle and is equal to the half of intercepted arc, that is to say, half of
\[\angle D'OB.\] This indicates that \(\angle D'OB\) is equal to \(2\varphi\). For the triangle AF'B, \(\angle AF'B = 90^\circ\), \(\angle F'AB = \beta_2 + \alpha\), therefore

\[F'B = AB \sin(\beta_2 + \alpha) = h_m \sin(\beta_2 + \alpha),\]  
(A1)

For the right triangle F'D'B, we have

\[D'B = \frac{F'B}{\cos \varphi} = \frac{h_m \sin(\beta_2 + \alpha)}{\cos \varphi},\]  
(A2)

Also, for the triangle OAB, \(\angle OAB = 180 - (\alpha + \beta_1)\), and \(\angle AOB = \beta_1 - \beta_2\), by using the Law of Sines,

\[OB = \frac{AB \sin(180 - \alpha - \beta_1)}{\sin(\beta_1 - \beta_2)} = \frac{h_m \sin(\alpha + \beta_1)}{\sin(\beta_1 - \beta_2)},\]  
(A3)

Because D'B is chord length and AB is radius of the fold circle, the value of D'B is

\[D'B = 2OB \sin \varphi = \frac{2h_m \sin(\alpha + \beta_1) \sin \varphi}{\sin(\beta_1 - \beta_2)},\]  
(A4)

By combining equations (A2) and (A4), the following result can be obtained

\[\sin (2\varphi) = \frac{\sin(\alpha + \beta_2) \sin(\beta_1 - \beta_2)}{\sin(\alpha + \beta_1)}\]  
(A5)

As a result, value of \(\varphi\) can be calculated by

\[\varphi = \frac{1}{2} \arcsin \frac{\sin(\alpha + \beta_2) \sin(\beta_1 - \beta_2)}{\sin(\alpha + \beta_1)},\]  
(A6)

For the triangle AD'B, \(\angle D'BA = 90 - \beta_2 - \alpha - \varphi\), \(\angle D'AB = \angle F'AB = \beta_2 + \alpha\), and \(\angle AD'B = 90 + \varphi\), by using the Law of Sines,

\[AD' = \frac{AB \sin(90 - \alpha - \beta_2 - \varphi)}{\sin(90 + \varphi)} = \frac{h_m \cos(\beta_2 + \alpha + \varphi)}{\cos(\varphi)},\]  
(A7)

Similarly, if the well azimuth is opposite to the azimuth of the bed [Figure 3(b)], the value of \(\varphi\) can be

\[\varphi = \frac{1}{2} \arcsin \frac{\sin(\alpha - \beta_2) \sin(\beta_2 - \beta_1)}{\sin(\alpha - \beta_1)}\]  
(A8)

As a result, the value of AD' is

\[AD' = \frac{AB \sin(90 - \alpha + \beta_2 - \varphi)}{\sin(90 + \varphi)} = \frac{h_m \cos(\alpha + \beta - \beta_2)}{\cos(\varphi)},\]  
(A9)

Figure 3(c) shows the case where the well azimuth is the same as the azimuth of the bed in an antiform. Likewise, we assume that the value of \(\angle D'BF'\) to be \(\varphi\). This angle is a tangent chord angle and is equal to the half of intercepted arc, that is to say, half of
\[ \angle D'OB. \text{ This indicates that } \angle D'OB \text{ is equal to } 2\varphi. \text{ For the triangle } AF'B, \angle AF'B = 90^\circ, \angle F'AB = \beta_1 + \alpha, \text{ therefore,} \]

\[ F'B = AB \sin (\beta_1 + \alpha) = h_m \sin (\beta_1 + \alpha), \quad \text{(A10)} \]

For the right triangle F'D'B, we have

\[ D'B = \frac{F'B}{\cos \varphi} = \frac{h_m \sin (\beta_1 + \alpha)}{\cos \varphi} \quad \text{(A11)} \]

On the other hand, for the triangle OAB, \( \angle OBA = 180 - (\alpha + \beta_2) \), and \( \angle AOB = \beta_2 - \beta_1 \), by using the Law of Sines,

\[ OB = \frac{AB \sin (180 - \alpha - \beta_2)}{\sin (\beta_1 - \beta_2)} = \frac{h_m \sin (\alpha + \beta_2)}{\sin (\beta_2 - \beta_1)} \quad \text{(A12)} \]

Because D'B is chord length and AB is radius of the fold circle, the value of D'B is

\[ D'B = 2OB \sin \varphi = \frac{2h_m \sin (\alpha + \beta_2) \sin \varphi}{\sin (\beta_2 - \beta_1)} \quad \text{(A13)} \]

By combining equations (A11) and (A13), the following result can be obtained

\[ \sin (2\varphi) = \sin (\beta_2 - \beta_1) \quad \text{(A14)} \]

As a result, value of \( \varphi \) can be calculated by

\[ \varphi = (\beta_2 - \beta_1)/2 \quad \text{(A15)} \]

For the triangle AD'B, \( \angle D'BA = 90 - \beta_2 - \alpha + \varphi \), \( \angle D'AB = \angle F'AB = \beta_2 + \alpha \), and \( \angle AD'B = 90 - \varphi \), by using the Law of Sines,

\[ AD' = \frac{AB \sin (90 - \alpha - \beta_2 + \varphi)}{\sin (90 - \varphi)} = \frac{h_m \cos (\beta_2 + \alpha - \varphi)}{\cos (\varphi)} \quad \text{(A16)} \]

Similarly, if the well azimuth is opposite to the azimuth of the bed [Figure 3(d)], the value of \( \varphi \) can be

\[ \varphi = (\beta_1 - \beta_2)/2 \quad \text{(A17)} \]

As a result, the value of AD' is

\[ AD' = \frac{AB \sin (90 - \alpha + \beta_2 + \varphi)}{\sin (90 - \varphi)} = \frac{h_m \cos (\alpha - \varphi - \beta_2)}{\cos (\varphi)} \quad \text{(A18)} \]